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TECHNICAL NOTE

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ELASTIC DESIGN CHARTS FOR THIN PLATES WITH SPANWISE AND CHORDWISE VARIATIONS IN TEMPERATURE

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SUMMARY

A set of graphs and tables is presented which permits the rapid determination of the elastic state of stress at any point in a thin, flat rectangular plate of uniform thickness loaded by both spanwise and chordwise thermal gradients. In addition, all relations from which the graphs and tables were developed and the derivation of those expressions are given. Examples illustrating the method of using the graphs and tables and demonstrating the accuracy of the results obtained therefrom are presented.

INTRODUCTION

Restrictions on the weight and size of many modern structures make necessary a very accurate estimate of the state of stress in a given application to take full advantage of the load-carrying capacity of available materials. It is frequently impossible to produce exact solutions for a specific geometry, and common practice is to approximate the exact configuration by one or a series of simple geometric shapes such as plates, cylinders, disks, and spheres. Even that simplification very often leaves a problem whose solution is of considerable difficulty.

The problem of determining the state of stress in a rectangular plate of uniform thickness subjected to a thermal gradient has been treated extensively in reference 1. However, that treatment is for chordwise temperature variations only, and completely neglects end effects. Other authors (refs. 2 to 6) have treated the problem in detail, taking into consideration end effects and two-dimensional temperature distributions. They make use of methods of various degrees of approximation, some of which are quite difficult to use.

The collocation procedure developed in reference 2 was shown to give accurate solutions to this problem. Its disadvantage is that it requires operating on a system of linear, ordinary differential equations whose solutions involve complex coefficients and exponents. If, however, the variables of the problem are properly dimensionalized, the system need be solved only once for all problems. Further, the availability of high-speed computing equipment simplifies the evaluation of the solution for a variety of conditions.

It is the purpose of this paper to present the solution to the problem of determining the elastic state of stress in a rectangular plate of uniform thickness subjected to a thermal gradient in two directions. The solution, which was developed by the collocation method, is in the form of tabulated functions that may be easily combined linearly to produce very accurately the elastic stress distribution for a large class of problems and for span-to-chord ratios varying between 0.1 and 3.0. The major restrictions are that the temperature distribution does not change rapidly between the stations at which the solution is developed and that the function representing the temperature distribution must be separable into the sum of a function in x only and a function in y only. Only solutions for the longitudinal and lateral stresses are presented. The analysis of reference 2 showed that the shearing stresses developed are extremely small.

The remainder of the main body of the report is devoted to an explanation and examples of the use of the solutions. Appendix A briefly outlines the general method of obtaining the solutions and presents literal expressions for them. A detailed derivation of the method of solution is given in reference 2.

METHOD OF USING TABLES AND CHARTS

To solve a specific problem, either tables I to XXVI or figures 1 to 26 may be used; the same information is presented in both. The information in the tables is more directly accessible, but the figures permit more rapid interpolation when such becomes necessary. The figures also quickly make apparent the location of the maximum stresses. Comparison of tables I and II with figures 1 and 2 will disclose the nature of the information presented therein.

It will be observed that the two tables and the two figures contain values of two sets of stress parameters, Θ_i and Φ_i, as a function of a pair of rectangular coordinates, x and y. These stress parameters are so defined that the stresses at each point may be determined from them as follows:

$$\sigma_x = E_o \alpha_o T_o (G_1 \Theta_1 + G_2 \Theta_2 + G_3 \Theta_3)$$

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and

$$\sigma_y = E_o\alpha_o T_o(G_1\Phi_1 + G_2\Phi_2 + G_3\Phi_3)$$

where the G_i depend on the temperature distribution for a given problem. (All symbols are defined in appendix B.) Furthermore, this information corresponds to a specific value of the span-to-chord ratio β, namely, $\beta = 0.1$. Examination of the remaining tables and figures reveals that there are two tables and two figures for each of thirteen values of β ranging from $\beta = 0.1$ to 3.0. Finally, it will be noticed that table I is labeled for "even temperature distributions" and table II for "odd temperature distributions." Similarly, figure 1 is for even, and figure 2 for odd, temperature distributions. All the tables and figures are divided in this manner.

The remainder of this section is devoted to an explanation of the use of the information in the tables. The same instructions apply unconditionally to the use of the figures.

In order to make use of the information tabulated therein, there must be available a function relating the product of Young's Modulus, coefficient of linear thermal expansion, and temperature ($E\alpha T$) with the coordinates of the plate (fig. 27). This may be either a literal expression or a numerical array. Once this function has been obtained, and the overall dimensions of the plate established, two separate procedures must be followed to yield the stresses at a given point within the plate. The first is labeled Preparatory Procedure, and the second is called Specific Calculations. They are outlined step by step in the ensuing discussion, and examples are shown in the following section.

Preparatory Procedure:

(1) Orient the plate with respect to a set of Cartesian coordinates so that the origin is at the center of the plate and there is a temperature variation in the vertical direction. For temperature variations in both directions the problem must be divided into two parts, with the plate rotated 90° for the second part and the entire procedure, steps 2 to 14, repeated.

(2) The maximum ordinate is arbitrarily defined as the semichord, and the maximum abscissa as the semispan. Divide all coordinates by the semichord dimension. The maximum ordinate is now unity. The maximum abscissa is defined to be β.

(3) Express the $E\alpha T$ function in terms of the x-y coordinates developed in step 2.

(4) Find the Laplacian of the $E\alpha T$ function. This Laplacian will be called the L function. In those cases for which there is available a literal expression for $E\alpha T$,

$$L = \nabla^2(E\alpha T) = \frac{\partial^2}{\partial x^2}(E\alpha T) + \frac{\partial^2}{\partial y^2}(E\alpha T)$$

In those cases for which $E\alpha T$ has been expressed as a numerical array, a finite difference approximation must be used. The following five-point central-difference formula is widely used:

$$\nabla^2(E\alpha T)_{ij} \cong \frac{1}{H^2}\Big[(E\alpha T)_{i-1,j-1} + (E\alpha T)_{i-1,j+1} + (E\alpha T)_{i+1,j-1} + (E\alpha T)_{i+1,j+1} - 4(E\alpha T)_{ij}\Big]$$

where H is the distance between adjacent points. It will be observed that the preceding expression is valid only for points not on the boundaries; but the subsequent steps will show that L is not needed on the boundaries. More accurate approximations are readily available from the literature and in some cases may be desirable. Finally, it may be found that performing the operation in step 8 before this step produces numbers that are easier to use in performing numerical calculations.

(5) Separate the L function into the sum of two functions, $L(x)$ and $L(y)$. An $L(x)$ will exist only if there is a temperature variation in both directions. Such a circumstance is handled by the procedure outlined in step 1. Therefore, only the $L(y)$ produced by this step is used in subsequent steps.

In general, performing such a separation is a difficult task. However, in practice the problem is considerably simplified. The most obvious method is to approximate the function as a polynomial or a Fourier series, taking care not to consider terms containing products of x and y. In the case for which L is an array, such a procedure is necessary.

(6) Separate $L(y)$ into the sum of two functions: an even function L_{even}, and an odd function L_{odd}. The following steps, 7 to 12, must be repeated for each, L_{even} and L_{odd}.

No complicated analytical method need be applied at this point because the value of the function is required at only three points. The simplest method is the following:

$$L_{even}(y_k) = \frac{1}{2}\left[L(y_k) + L(-y_k)\right]$$

$$L_{odd}(y_k) = \frac{1}{2}\left[L(y_k) - L(-y_k)\right]$$

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(7) Evaluate L_{even} at $y = 1/6$, $y = 3/6$, and $y = 5/6$.

(8) Divide the three values found in step 7 by the product of the reference data, $E_o\alpha_oT_o$. The three numbers produced are G_1, G_2, and G_3, corresponding to $y = 1/6$, $3/6$, and $5/6$, respectively.

(9) Choose the table corresponding to the value of β produced in step 2. Use the even or odd table, depending on whether L_{even} or L_{odd} was used to produce the G_i in step 8.

Specific Calculations:

(10) Choose a value of x and a value of y at which the stresses are desired.

(11) Read Θ_i and Φ_i from the table for that value of x and y.

(12) Calculate $\Theta = \sum_{i=1}^{3} G_i\Theta_i$ and $\Phi = \sum_{i=1}^{3} G_i\Phi_i$.

(13) Sum the Θ's and Φ's found for L_{even} and L_{odd} at each value of the coordinates. Care must be exercised to attach the proper sign to L_{even} and L_{odd} for quadrants other than the first.

(14) The stresses at each point for the component of thermal gradient being calculated for this orientation of the plate are products of the summed stress parameters at that point and the value of $E_o\alpha_oT_o$ at the reference point: $\sigma_x = E_o\alpha_oT_o\Theta$ and $\sigma_y = E_o\alpha_oT_o\Phi$.

(15) For the case in which there is temperature variation in both directions, another set of stresses must be added to the set found in step 14. Extreme care must be exercised at this point to insure that the stresses summed all correspond to the same point in the plate and that the proper stresses are summed. The definition of lateral and longitudinal stresses changes when the plate is rotated. The tables and graphs are given in terms of x/β ranging from 0 to 1 rather than in terms of x ranging between 0 and β. This is a considerable aid in keeping track of corresponding points in the two separate calculations that must be performed for the temperature variations in each direction.

EXAMPLES

As a first example, consider the stress distribution in an infinitely long, flat strip of uniform thickness produced by a parabolic-chordwise thermal gradient defined by $T/T_o = y^2 - 1/3$, E and α being constant. The method of reference 1 produces the following stresses at a distance far from the free end:

$$\Theta = \frac{\sigma_x}{E\alpha T_o} = \frac{1}{3} - y^2$$

$$\Phi = \frac{\sigma_y}{E\alpha T_o} = 0$$

Using equations (A18) and (A19) gives

$$G_i(y_i) = \nabla^2(T(y_i)/T_o) = \nabla^2\left(y^2 - \frac{1}{3}\right) = 2$$

$$\Theta(x,y) = \sum_{i=1}^{3} 2\Theta_i(x,y)$$

$$\Phi(x,y) = \sum_{i=1}^{3} 2\Phi_i(x,y)$$

With the assumption that the solution given at $x = 0$ for $\beta = 3$ approximates the solution for the semi-infinite plate, table XXV was used for the values of Θ_i and Φ_i; $\Theta(0,0)$, for example, is given as

$$\Theta(0,0) = 2(0.1217 + 0.0414 + 0.0044) = 0.335$$

Table XXXIII and figure 28 compare the stresses calculated by the two methods. It is to be noted that the maximum difference in Θ is quite small - an amount not discernible on the graph.

As a second example, consider a plate having a span-to-chord ratio of 2 and subjected to a thermal gradient defined by

$$\frac{T}{T_o} = \sin(5x) + \cos(y)$$

$$G_i = \nabla^2\left(\frac{T}{T_o}\right) = -25 \sin(5x) - \cos(y)$$

The problem will be divided into two parts, and the results from each summed. First, consider the variation in the x direction; β has been defined as the ratio of span to chord for chordwise temperature variations. So, for this case, the plate must be so oriented that the shortest dimension lies parallel to the x axis, and $\beta = 0.5$, $G_i = -25 \sin(5y_i)$. Sin(y) is an odd function; therefore, table X is used for the values of Θ_i and Φ_i. The values of G_i are $G_1 = -18.50$, $G_2 = -14.96$, and $G_3 = 21.37$. For the thermal gradient in the y

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direction, the plate is rotated 90°. Now, $\beta = 2.0$, $G_i = -\cos(y_i)$, and, $\cos(y)$ being an even function, table XXI is used for the values of Θ_i and Φ_i. The G_i are $G_1 = -0.9861$, $G_2 = -0.8776$, and $G_3 = -0.6724$. Figures 29 and 30 show the results of the calculations along the x and y axes.

As a final example, consider a plate having a span-to-chord ratio of 3 and the following temperature distribution:

$$\frac{T}{T_o} = 1.014 - 0.08940\,y + 0.001074\,(1 - y)^9$$

This problem is also considered in reference 6, using the methods of references 3 and 4. Figures 31 and 32 compare the results obtained by using those methods with the results of this report. Figure 33 compares with the results of this report the solution far from the end of an infinitely long strip, which closely approximates the solution for large values of β.

DISCUSSION

The examples of the preceding section demonstrate the accuracy of the collocation method for thermal stresses in flat plates. Where the stresses can be found by the exact method described in reference 1, there is virtually no difference between the exact solution and the approximate solution presented in this report. Figures 28 and 33 show no discernible difference for the parabolic and the ninth-degree temperature distributions. Table XXXIII shows the solution to more significant digits than can be plotted on a graph the size of the pages in this report, and indicates a very small maximum difference for the parabolic distribution.

Figures 31 and 32 compare certain results of the collocation method with the results of the energy method of Heldenfels and Roberts (ref. 3) and the method of self-equilibrating polynomials of Horvay (ref. 4). There is quite close agreement in the dimensionless longitudinal stresses Θ among the three methods. In fact, there is no discernible difference between the collocation method and the method of self-equilibrating polynomials. The latter two methods also show close agreement in the lateral stresses.

Figure 32 indicates much poorer agreement between the maximum chordwise stress calculated by the method of reference 3 and the other two methods. However, it should be noted that in magnitude this difference is no larger than the greatest difference for the spanwise stress.

The results plotted in figures 31 and 32 are the result of calculations based on equations (A20) of appendix A. Extreme caution must be exercised in interpolating between the values plotted in the figures. For this example, it was found necessary to cross-plot the proper figures and use a quadratic interpolation formula because the curves are varying quite rapidly at $y = 0.9$. Results obtained from attempts to interpolate by eye directly from the graphs were very poor, and linear interpolation after cross-plotting gave only fair answers. The quadratic interpolation gave much more satisfactory answers.

The large number of calculations necessary for the production of the graphs and tables for this paper makes essential a rigorous checking procedure for any confidence to be placed in the results. In addition to using normal checking procedures, the entire problem was solved a second time using finite difference methods. The solution produced was the algebraic sum of the Θ_i and the algebraic sum of the Φ_i at each point. Excellent correlation was obtained between the two methods, giving confidence not only in the correctness of the calculations but also in the accuracy of the method.

It should be noted that the equations solved for this report, equation (A4) with boundary conditions (A5) and (A6), describe other physical situations than that treated here. Therefore, the charts and tables presented can be used for any problem that is defined by equations (A4) to (A6). For example, the familar problem of determining the deflections in a rectangular plate rigidly clamped along all four edges is in this category. The equation to be solved is

$$\nabla^4 W\ (x,y) = \frac{1}{D}\, q\ (x,y) \tag{1}$$

where W is the deflection, q the load, and D the flexural rigidity, defined by

$$D = \frac{Eh^3}{12(1 - \nu^2)} \tag{2}$$

where h is the thickness, and E and ν are Young's modulus and Poisson's ratio, respectively. The maximum moments are given by

$$M_x = -D\left[\frac{1}{\beta^2}\frac{\partial^2 W}{\partial x^2} + \nu\,\frac{\partial^2 W}{\partial y^2}\right], \qquad M_y = -D\left[\frac{\partial^2 W}{\partial y^2} + \frac{\nu}{\beta^2}\frac{\partial^2 W}{\partial x^2}\right] \tag{3}$$

and the stresses at the outer fibers are

$$\sigma_x = \frac{6M_x}{h^2}, \qquad \sigma_y = \frac{6M_y}{h^2} \tag{4}$$

It must be borne in mind that the tables here are in terms of dimensionless quantities obtained by dividing the equations through by the right-hand side. So, for the preceding problem, the tables can be considered as giving the following quantities:

$$\Theta = \frac{\partial^2}{\partial y^2} \frac{WD}{q}, \qquad \Phi = \frac{1}{\beta^2} \frac{\partial^2}{\partial x^2} \frac{WD}{q} \tag{5}$$

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APPENDIX A

ANALYSIS

The problem of defining the elastic stress distribution in a thin flat plate falls into the category of plane stress problems. The equation defining such problems is quite frequently written in terms of Airy's stress function, the stresses in turn being defined in terms of that function (ref. 1). Using that procedure for the thin flat plate subjected to a thermal gradient produces the partial differential equation (ref. 2)

$$\nabla^4 W(\xi,\eta) = -\nabla^2 E(\xi,\eta)\alpha(\xi,\eta)T(\xi,\eta) \tag{A1}$$

with the boundary conditions

$$\left.\begin{aligned} W(b,\eta) &= 0 \\ \frac{\partial}{\partial\xi} W(b,\eta) &= 0 \\ W(\xi,a) &= 0 \\ \frac{\partial}{\partial\eta} W(\xi,a) &= 0 \end{aligned}\right\} \tag{A2}$$

derived from the normal and shear stresses vanishing on the boundary, and where the stresses are defined by

$$\left.\begin{aligned} \sigma_\xi &= \frac{\partial^2}{\partial\eta^2} W(\xi,\eta) \\ \sigma_\eta &= \frac{\partial^2}{\partial\xi^2} W(\xi,\eta) \\ \tau_{\xi\eta} &= -\frac{\partial^2}{\partial\xi\,\partial\eta} W(\xi,\eta) \end{aligned}\right\} \tag{A3}$$

The solution to equations (A1), (A2), and (A3) is presented in detail in reference 2. The method is summarized in the remainder of this section with the nomenclature of this paper.

For greatest generality, a transformation is made to another coordinate system having the following properties:

$$x = \xi/a; \qquad y = \eta/a; \qquad \beta = b/a$$

The plate is shown with respect to these coordinate systems in figure 27. Rewriting in the upper-right quadrant of the new coordinate system, equation (A1) becomes

$$\nabla^4 \Omega(x,y) = -\nabla^2 \Gamma(x,y) \tag{A4}$$

where

$$\Gamma = \frac{E\alpha T}{E_o \alpha_o T_o}$$

and Ω is Airy's stress function in the new coordinate system, and equations (A2) and (A3) become

$$\Omega(\beta,y) = \frac{\partial}{\partial x}\Omega(\beta,y) = \Omega(x,1) = \frac{\partial}{\partial y}\Omega(x,1) = 0 \tag{A5}$$

$$\left.\begin{aligned}
\Theta &= \frac{\sigma_x}{E_o \alpha_o T_o} = \frac{\partial^2}{\partial y^2}\Omega(x,y) \\
\Phi &= \frac{\sigma_y}{E_o \alpha_o T_o} = \frac{\partial^2}{\partial x^2}\Omega(x,y) \\
\lambda &= \frac{\tau_{xy}}{E_o \alpha_o T_o} = \frac{-\partial^2}{\partial x\,\partial y}\Omega(x,y)
\end{aligned}\right\} \tag{A6}$$

For variations in both the x and the y directions, two separate solutions can be found and superposed, inasmuch as the differential equation (A1) is linear and the principle of superposition applies. For simplicity, then, assume that the temperature varies only in the y direction:

$$T = T(y), \qquad \Gamma = \Gamma(y) \tag{A7}$$

For temperature variations in the other direction, the obvious coordinate transform may be made. For additional simplicity, with no loss of generality, separate T into two functions

$$T(y) = T_{even}(y) + T_{odd}(y) \tag{A8}$$

such that T_{even} is an even function and T_{odd} an odd function. Again by superposition, two solutions can be found and combined. In the ensuing discussion only the even part is treated. The odd part is handled in a similar manner.

It should be noted that nowhere in the following discussion is any restriction placed on E and α. These quantities also may vary with the coordinates of the plate.

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Solving equation (A4) approximately, using a collocation procedure whereby the differential equation is satisfied everywhere in x but only at a finite number of equally spaced values of y, the solution is assumed to be of the form:

$$\Omega(x,y) = \sum_{k=1}^{n} \Omega_k(x)\ P_k(y) \tag{A9}$$

where P_k is a polynomial associated with the k^{th} station and has the property

$$P_k(y_j) = \delta_{kj} \tag{A10}$$

and satisfies the boundary conditions in y, namely:

$$P_k(1) = P_k'(1) = 0 \tag{A11}$$

Such polynomials are easily obtainable and for the even temperature distribution have the form:

$$P_k(y) = \frac{(y^2 - 1)^2}{(y_k^2 - 1)^2} \prod_{j \neq k} (y^2 - y_j^2) \Big/ \prod_{j \neq k} (y_k^2 - y_j^2),\quad 0 < y_k < 1 \tag{A12}$$

Evaluating equations (A9) and (A12) and substituting into equation (A4) at each station produce a system of n simultaneous, linear, ordinary differential equations of the form

$$\left.\begin{aligned}
&\Omega_1'''' + D_{11}\Omega_1'' + E_{11}\Omega_1 \\
&\quad + D_{21}\Omega_2'' + E_{21}\Omega_2 \\
&\quad + \cdots \\
&\quad + D_{n1}\Omega_n'' + E_{n1}\Omega_n = -\nabla^2\Gamma(y_1) \\
&\cdots \\
&\Omega_n'''' + D_{1n}\Omega_1'' + E_{1n}\Omega_1 \\
&\quad + D_{2n}\Omega_2'' + E_{2n}\Omega_2 \\
&\quad + \cdots \\
&\quad + D_{nn}\Omega_n'' + E_{nn}\Omega_n = -\nabla^2\Gamma(y_n)
\end{aligned}\right\} \tag{A13}$$

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where the D_{ij} and E_{ij} are defined as follows:

$$\left.\begin{aligned} D_{ij} &= 2P_i''(y_j) \\ E_{ij} &= P_i''''(y_j) \end{aligned}\right\} \qquad \text{(A14)}$$

Inasmuch as the right-hand sides are even functions of x (they are actually constant with respect to x), only the even part of the solution is necessary. Using conventional procedures produces the homogeneous solution as a sum of exponential quantities. Rearranging for convenience produces the following solution:

$$\Omega_i = \sum_{j=1}^{n} \left\{ A_j \left[a_{ij} \cosh(g_j x) \cos(h_j x) + b_{ij} \sinh(g_j x) \sin(h_j x) \right] \right.$$
$$\left. + B_j \left[-b_{ij} \cosh(g_j x) \cos(h_j x) + a_{ij} \sinh(g_j x) \sin(h_j x) \right] \right\} \qquad \text{(A15)}$$

$$i = 1, 2, \ldots$$

where the g_j and h_j are solutions of the determinantal equation associated with system (A13).

In order to determine the constants A_j and B_j and the particular solutions, and hence the complete solutions, the specific values of the right-hand sides must be known. Since the right-hand sides are constant, a particular solution may be formed from a linear combination of the right-hand sides; that is,

$$\Omega_i^{(p)} = \sum_{j=1}^{n} l_{ij} \nabla^2 T(y_j) \qquad \text{(A16)}$$

where the l_{ij} are solutions of a set of n linear algebraic equations resulting from the substitution of equations (A16) into equations (A13). Once the particular solutions have been obtained, they may be combined with the homogeneous solutions, equations (A15), and substituted into the expressions for the boundary conditions, equations (A5), developing a system of $2n$ linear algebraic equations in A_j and B_j.

It is desirable to develop a right-hand side to system (A13) and a solution for it in such a manner that the complete solution to any specific problem may be rapidly determined from that solution, obviating the necessity for the development of, and solutions to, systems of algebraic equations. Because system (A13) is linear, such a procedure may be followed.

A set of n right-hand sides for system (A13) was chosen, and n solutions were found. These n solutions may be combined for any specific problem, the combination depending on the temperature distribution for that problem. Each right-hand side is the negative of one of the columns of the identity matrix of order n. This results in n solutions of the form of equation (A15), differing only in the values of A_j and B_j and n sets of particular solutions associated with them. Substitution in equation (A6) produces the following expressions for the stresses:

$$\left.\begin{aligned}
\Theta(x,y) &= \sum_{i=1}^{n} F_i(y_i) \sum_{j=1}^{n} \Omega_{ij}(x)\ P_j''(y) \\
\Phi(x,y) &= \sum_{i=1}^{n} F_i(y_i) \sum_{j=1}^{n} \Omega_{ij}''(x)\ P_j(y) \\
\lambda(x,y) &= -\sum_{i=1}^{n} F_i(y_i) \sum_{j=1}^{n} \Omega_{ij}'(x)\ P_j'(y)
\end{aligned}\right\} \tag{A17}$$

where

$$F_i(y_i) = \nabla^2 T(y_i)$$

It was shown in reference 2 that, for problems for which the exact solution to equation (A1) could be found, the preceding method gives extremely accurate results, and no substantial increase in accuracy resulted from increasing n above 3. Therefore, all further results given in this paper are for $n = 3$.

Solutions

The relations that follow describe the solutions for either even or odd temperature gradients, depending on the values assigned the coefficients. Tables XXVII to XXXII tabulate all the coefficients necessary to define completely the solutions except for the multiplicative constants G_i, which depend on the temperature distribution. The constants A_{jk} and B_{jk} depend on the boundary conditions and hence the value of β. For values of β other than those tabulated, they must be computed from the boundary conditions in the normal manner. The solutions are as follows:

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$$\left.\begin{aligned}
\Theta(x,y) &= \sum_{i=1}^{3} G_i(y_i)\ \Theta_i(x,y) \\
\Phi(x,y) &= \sum_{i=1}^{3} G_i(y_i)\ \Phi_i(x,y) \\
\lambda(x,y) &= \sum_{i=1}^{3} G_i(y_i)\ \lambda_i(x,y)
\end{aligned}\right\} \tag{A18}$$

where

$$G_i(y_i) = \nabla^2\Gamma(y_i) \tag{A19}$$

and

$$\left.\begin{aligned}
\Theta_i(x,y) &= \sum_{j=1}^{3} \Omega_{ij}(x)\ P_j''(y) \\
\Phi_i(x,y) &= \sum_{j=1}^{3} \Omega_{ij}''(x)\ P_j(y) \\
\lambda_i(x,y) &= -\sum_{j=1}^{3} \Omega_{ij}'(x)\ P_j'(y)
\end{aligned}\right\} \tag{A20}$$

where $i = 1, 2, 3.$

Note that, of the following systems of equations, systems (A21) and (A23) have slightly differing forms for even and odd temperature distributions. This is because the determinantal equation associated with the even temperature distribution has all complex solutions, while the determinantal equation for the odd distribution has two real solutions.

$$\left.\begin{aligned}
\Omega_{ij} &= \sum_{k=1}^{3} \left\{A_{jk}\left[a_{ik}\omega_k^{(1)} + b_{ik}\omega_k^{(2)}\right] + B_{jk}\left[-b_{ik}\omega_k^{(1)} + a_{ik}\omega_k^{(2)}\right]\right\} + \Omega_{ij}^{(p)} \\
\Omega_{ij}' &= \sum_{k=1}^{3} \left\{A_{jk}\left[c_{ik}\omega_k^{(3)} + d_{ik}\omega_k^{(4)}\right] + B_{jk}\left[-d_{ik}\omega_k^{(3)} + c_{ik}\omega_k^{(4)}\right]\right\} \\
\Omega_{ij}'' &= \sum_{k=1}^{3} \left\{A_{jk}\left[e_{ik}\omega_k^{(1)} + f_{ik}\omega_k^{(2)}\right] + B_{jk}\left[-f_{ik}\omega_k^{(1)} + e_{ik}\omega_k^{(2)}\right]\right\}
\end{aligned}\right\} \tag{A21a}$$

where $i = 1, 2, 3;\ j = 1, 2, 3;$ even case only.

$$\left.\begin{aligned}
\Omega_{ij} &= \sum_{k=1}^{2} \left\{A_{jk}\left[a_{ik}\omega_k^{(1)} + b_{ik}\omega_k^{(2)}\right] + B_{jk}\left[-b_{ik}\omega_k^{(1)} + a_{ik}\omega_k^{(2)}\right]\right\} \\
&\qquad + A_{j3}a_{i3}\omega_3^{(1)} + B_{j3}b_{i3}\omega_3^{(2)} + \Omega_{ij}^{(p)} \\
\Omega'_{ij} &= \sum_{k=1}^{2} \left\{A_{jk}\left[c_{ik}\omega_k^{(3)} + d_{ik}\omega_k^{(4)}\right] + B_{jk}\left[-d_{ik}\omega_k^{(3)} + c_{ik}\omega_k^{(4)}\right]\right\} \\
&\qquad + A_{j3}c_{i3}\omega_3^{(3)} + B_{j3}d_{i3}\omega_3^{(4)} \\
\Omega''_{ij} &= \sum_{k=1}^{2} \left\{A_{jk}\left[e_{ik}\omega_k^{(1)} + f_{ik}\omega_k^{(2)}\right] + B_{jk}\left[-f_{ik}\omega_k^{(1)} + e_{ik}\omega_k^{(2)}\right]\right\} \\
&\qquad + A_{j3}e_{i3}\omega_3^{(1)} + B_{j3}f_{i3}\omega_3^{(2)}
\end{aligned}\right\} \quad \text{(A21b)}$$

where $i = 1, 2, 3$; $j = 1, 2, 3$; odd case only.

$$\left.\begin{aligned}
\omega_k^{(1)} &= \cosh(g_k x)\ \cos(h_k x) \\
\omega_k^{(2)} &= \sinh(g_k x)\ \sin(h_k x) \\
\omega_k^{(3)} &= \sinh(g_k x)\ \cos(h_k x) \\
\omega_k^{(4)} &= \cosh(g_k x)\ \sin(h_k x) \\
&\text{for even case: } k = 1, 2, 3; \text{ for odd case: } k = 1, 2. \\
\omega_3^{(1)} &= \cosh(g_3 x) \\
\omega_3^{(2)} &= \cosh(h_3 x) \\
\omega_3^{(3)} &= \sinh(g_3 x) \\
\omega_3^{(4)} &= \sinh(h_3 x)
\end{aligned}\right\} \quad \text{(A22)}$$

for odd case only.

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$$\left.\begin{aligned} P_j(y) &= \sum_{k=1}^{5} r_{jk}\, y^{2k-2} \\ P'_j(y) &= \sum_{k=1}^{4} s_{jk}\, y^{2k-1} \\ P''_j(y) &= \sum_{k=1}^{4} t_{jk} y^{2k-2} \end{aligned}\right\} \tag{A23a}$$

where $j = 1, 2, 3$; even case only.

$$\left.\begin{aligned} P_j(y) &= \sum_{k=1}^{5} r_{jk}\, y^{2k-1} \\ P'_j(y) &= \sum_{k=1}^{5} s_{jk}\, y^{2k-2} \\ P''_j(y) &= \sum_{k=1}^{4} t_{jk}\, y^{2k-1} \end{aligned}\right\} \tag{A23b}$$

where $j = 1, 2, 3$; odd case only.

$$\Omega_{ij}^{(p)} = \ell_{ij}$$

where $i = 1, 2, 3$; $j = 1, 2, 3$.

The expressions for Θ_i and Φ_i from equations (A20) have been evaluated at a number of points for several values of β, and the solutions are tabulated in tables I to XXVI and are presented graphically in figures 1 to 26. The λ_i are not tabulated because reference 2 shows the shearing stresses to be quite small in relation to the other stresses for problems of this type. Cross plotting of the graphs enables one to obtain solutions at intermediate values of β, and permits more accurate interpolation between values of y. Those solutions, used in equations (A17) and (A18), give the stress distribution in any plate having a span-to-chord ratio of β. For two-dimensional thermal gradients the plate is oriented one way with respect to the x,y axes, and the solution is found for the thermal gradient in the y direction; then it is rotated 90°, and the solution is found for the reciprocal value of β and the new thermal gradient in the y direction. The stress at any point is the algebraic sum of the two stresses found.

Similarly, problems involving gradients that are neither odd nor even can be separated into two problems, one odd and the other even, and the stresses found summed.

Finally, it was found unnecessary to evaluate the solutions at values of β greater than 3. Such values result in the solution for the semi-infinite plate; hence, infinity can be regarded as 3 for this problem. It is to be noted that, for the larger values of β, the solutions are virtually identical.

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APPENDIX B

SYMBOLS

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A_j, A_{ij}, B_j, B_{ij}	coefficients of homogeneous solutions to differential equation for stress function
a,b	semichord and semispan dimensions of plate, respectively
a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}, f_{ij}, g_{ij}, h_{ij}	coefficients of functions in homogeneous solutions to differential equation for stress function
l_{ij}	coefficients of functions in particular solutions to differential equation for stress function
E	Young's modulus
E_o	reference value of Young's modulus
F_i, G_i	combining coefficients for complete stress at any point
H	mesh spacing in finite difference grid
L	Laplacian of $E\alpha T$
P_k	polynomial associated with k^{th} station function in y
r_{ij}, s_{ij}, t_{ij}	coefficients of polynomial associated with i^{th} station
T	temperature function
T_o	reference temperature
T_{even}	even portion of temperature function

T_{odd} odd portion of temperature function

W Airy's stress function in ξ, η coordinate system

x, y dimensionless spanwise and chordwise coordinates, respectively

y_j j^{th} station

α coefficient of linear thermal expansion

α_o coefficient of linear thermal expansion at reference temperature

β ratio of span to chord

Γ $\dfrac{E\alpha T}{E_o \alpha_o T_o}$

δ_{kj} Kroniker's delta, $\delta_{kj} = \begin{cases} 1, & k=j \\ 0, & k \neq j \end{cases}$

η, ξ chordwise and spanwise coordinates, respectively

Θ, Θ_i dimensionless spanwise stress

λ, λ_i dimensionless shear stress

σ_x, σ_ξ longitudinal stress

σ_y, σ_η lateral stress

$\tau_{xy}, \tau_{\xi\eta}$ shear stress

Φ, Φ_i dimensionless chordwise stress

$\left.\begin{matrix} \Omega, \Omega_i, \\ \Omega_{ij}, \Omega^{(p)} \end{matrix}\right\}$ dimensionless Airy's stress function

$\omega_k^{(j)}$ solution function to differential equations for stress

$\prod\limits_{j \neq k}$ product for all values of j except j=k

∇^2 Laplacian operator, $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

∇^4 biharmonic operator, $\dfrac{\partial^4}{\partial x^4} + 2\,\dfrac{\partial^4}{\partial x^2\,\partial y^2} + \dfrac{\partial^4}{\partial y^4}$

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TABLE I. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.10

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.0710	-0.1008	0.0700	1.988	-0.451	0.306
	0.2	0.0356	-0.0325	-0.0099	1.476	0.244	-0.121
	0.4	-0.0324	0.0868	-0.1117	0.447	1.379	-0.350
	0.6	-0.0505	0.0807	0.0145	-0.146	1.330	0.839
	0.8	0.0100	-0.0806	0.2415	-0.055	0.240	1.585
	1.0	-0.0373	0.1176	-0.7510	0.000	0.000	-0.000
0.2	0.	0.0655	-0.0930	0.0647	1.753	-0.403	0.273
	0.2	0.0328	-0.0300	-0.0091	1.301	0.214	-0.107
	0.4	-0.0299	0.0802	-0.1032	0.391	1.219	-0.314
	0.6	-0.0466	0.0744	0.0133	-0.130	1.172	0.741
	0.8	0.0093	-0.0746	0.2230	-0.047	0.206	1.406
	1.0	-0.0345	0.1090	-0.6935	0.000	0.000	-0.000
0.4	0.	0.0503	-0.0716	0.0500	1.045	-0.252	0.173
	0.2	0.0252	-0.0230	-0.0069	0.772	0.123	-0.066
	0.4	-0.0230	0.0618	-0.0796	0.226	0.733	-0.202
	0.6	-0.0358	0.0571	0.0101	-0.081	0.697	0.444
	0.8	0.0073	-0.0576	0.1718	-0.026	0.110	0.860
	1.0	-0.0269	0.0849	-0.5340	0.000	0.000	-0.000
0.6	0.	0.0293	-0.0419	0.0294	-0.145	0.014	-0.006
	0.2	0.0146	-0.0134	-0.0040	-0.112	-0.024	0.006
	0.4	-0.0135	0.0362	-0.0467	-0.043	-0.090	0.003
	0.6	-0.0208	0.0333	0.0058	0.005	-0.099	-0.057
	0.8	0.0044	-0.0340	0.1007	0.007	-0.037	-0.082
	1.0	-0.0160	0.0508	-0.3129	-0.000	-0.000	0.000
0.8	0.	0.0094	-0.0134	0.0095	-1.833	0.421	-0.286
	0.2	0.0047	-0.0043	-0.0013	-1.360	-0.223	0.112
	0.4	-0.0043	0.0116	-0.0150	-0.409	-1.275	0.329
	0.6	-0.0066	0.0106	0.0018	0.136	-1.225	-0.774
	0.8	0.0015	-0.0110	0.0323	0.050	-0.216	-1.470
	1.0	-0.0053	0.0167	-0.1003	-0.000	-0.000	0.000
1.0	0.	0.0000	-0.0000	0.0000	-4.040	1.005	-0.695
	0.2	0.0000	-0.0000	0.0000	-2.979	-0.469	0.259
	0.4	-0.0000	0.0000	-0.0000	-0.859	-2.852	0.818
	0.6	-0.0000	0.0000	-0.0000	0.321	-2.691	-1.722
	0.8	0.0000	-0.0000	0.0000	0.095	-0.396	-3.376
	1.0	-0.0000	0.0000	-0.0000	-0.000	-0.000	0.000

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TABLE II. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.10

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	-0.	0.	0.	-0.	0.	0.
	0.2	0.1755	-0.0694	0.0138	1.639	0.156	-0.046
	0.4	0.0589	0.0350	-0.0659	0.938	1.137	-0.149
	0.6	-0.1521	0.1354	-0.0332	-0.550	1.559	0.632
	0.8	0.0247	-0.0871	0.2464	-0.229	0.342	1.491
	1.0	-0.2368	0.2439	-0.8729	0.000	0.000	-0.000
0.2	0.	-0.	0.	0.	-0.	0.	0.
	0.2	0.1620	-0.0641	0.0128	1.450	0.134	-0.040
	0.4	0.0543	0.0323	-0.0608	0.827	1.003	-0.135
	0.6	-0.1405	0.1250	-0.0307	-0.488	1.376	0.557
	0.8	0.0230	-0.0806	0.2276	-0.199	0.296	1.324
	1.0	-0.2192	0.2260	-0.8063	0.000	0.000	-0.000
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	0.1245	-0.0495	0.0100	0.876	0.072	-0.023
	0.4	0.0416	0.0249	-0.0469	0.493	0.600	-0.090
	0.6	-0.1081	0.0962	-0.0239	-0.299	0.822	0.331
	0.8	0.0181	-0.0625	0.1755	-0.111	0.161	0.814
	1.0	-0.1700	0.1758	-0.6216	0.000	0.000	-0.000
0.6	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	0.0728	-0.0291	0.0060	-0.101	-0.024	0.005
	0.4	0.0241	0.0146	-0.0275	-0.069	-0.081	-0.004
	0.6	-0.0633	0.0563	-0.0143	0.027	-0.111	-0.048
	0.8	0.0110	-0.0370	0.1031	0.029	-0.049	-0.072
	1.0	-0.1008	0.1047	-0.3650	-0.000	-0.000	0.000
0.8	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	0.0233	-0.0093	0.0020	-1.516	-0.140	0.042
	0.4	0.0076	0.0047	-0.0088	-0.865	-1.049	0.141
	0.6	-0.0203	0.0180	-0.0047	0.511	-1.439	-0.582
	0.8	0.0037	-0.0120	0.0331	0.209	-0.309	-1.385
	1.0	-0.0329	0.0344	-0.1174	-0.000	-0.000	0.000
1.0	0.	0.	0.	0.	0.	-0.	-0.
	0.2	0.0000	-0.0000	0.0000	-3.416	-0.258	0.085
	0.4	-0.0000	-0.0000	0.0000	-1.904	-2.322	0.370
	0.6	-0.0000	0.0000	0.0000	1.179	-3.185	-1.274
	0.8	0.0000	-0.0000	0.0000	0.408	-0.584	-3.204
	1.0	-0.0000	0.0000	-0.0000	-0.000	-0.000	0.000

TABLE III. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.20

x/β	y	$\Theta_1\times10^3$	$\Theta_2\times10^3$	$\Theta_3\times10^3$	$\Phi_1\times10^3$	$\Phi_2\times10^3$	$\Phi_3\times10^3$
0.	0.	0.914	-1.143	0.481	7.16	-0.65	0.36
	0.2	0.497	-0.422	-0.154	5.55	1.24	-0.23
	0.4	-0.336	0.912	-0.907	2.15	4.53	-0.15
	0.6	-0.666	1.070	0.280	-0.19	4.86	2.46
	0.8	-0.042	-0.584	2.149	-0.30	1.70	3.58
	1.0	-0.116	-0.006	-6.901	0.00	0.00	-0.00
0.2	0.	0.845	-1.060	0.450	6.35	-0.63	0.35
	0.2	0.459	-0.390	-0.142	4.91	1.07	-0.21
	0.4	-0.312	0.847	-0.844	1.87	4.04	-0.19
	0.6	-0.616	0.989	0.258	-0.19	4.30	2.20
	0.8	-0.037	-0.547	1.997	-0.26	1.47	3.26
	1.0	-0.112	0.017	-6.411	0.00	0.00	-0.00
0.4	0.	0.654	-0.825	0.359	3.86	-0.53	0.29
	0.2	0.354	-0.302	-0.109	2.95	0.60	-0.16
	0.4	-0.244	0.662	-0.667	1.06	2.52	-0.25
	0.6	-0.476	0.764	0.196	-0.17	2.61	1.39
	0.8	-0.023	-0.439	1.569	-0.15	0.78	2.22
	1.0	-0.097	0.066	-5.026	0.00	0.00	-0.00
0.6	0.	0.386	-0.493	0.224	-0.40	-0.19	0.11
	0.2	0.208	-0.178	-0.063	-0.36	-0.16	-0.03
	0.4	-0.147	0.398	-0.408	-0.25	-0.15	-0.21
	0.6	-0.281	0.450	0.112	-0.08	-0.28	-0.06
	0.8	-0.008	-0.275	0.950	0.03	-0.28	0.16
	1.0	-0.068	0.091	-3.034	-0.00	0.00	-0.00
0.8	0.	0.125	-0.162	0.078	-6.63	0.66	-0.36
	0.2	0.067	-0.058	-0.020	-5.13	-1.12	0.22
	0.4	-0.049	0.132	-0.138	-1.96	-4.22	0.19
	0.6	-0.091	0.146	0.035	0.20	-4.50	-2.30
	0.8	-0.000	-0.096	0.319	0.28	-1.54	-3.40
	1.0	-0.027	0.054	-1.013	-0.00	-0.00	0.00
1.0	0.	0.000	0.000	0.000	-15.14	2.46	-1.40
	0.2	0.000	-0.000	0.000	-11.48	-2.21	0.68
	0.4	-0.000	-0.000	-0.000	-3.94	-10.05	1.36
	0.6	-0.000	0.000	-0.000	0.80	-10.22	-5.54
	0.8	0.000	0.000	0.000	0.55	-2.71	-9.29
	1.0	-0.000	-0.000	-0.000	-0.00	-0.00	0.00

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TABLE IV. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.20

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	-0.	0.	0.	-0.	0.	0.
	0.2	2.049	-0.657	-0.026	4.90	1.17	-0.14
	0.4	0.864	0.397	-0.560	3.22	4.10	0.11
	0.6	-1.651	1.546	-0.056	-1.32	5.46	2.02
	0.8	-0.164	-0.518	2.099	-1.15	2.14	3.27
	1.0	-1.337	0.724	-7.498	0.00	0.00	-0.00
0.2	0.	-0.	0.	0.	-0.	0.	0.
	0.2	1.899	-0.611	-0.021	4.40	1.01	-0.13
	0.4	0.797	0.368	-0.521	2.87	3.63	0.07
	0.6	-1.532	1.433	-0.056	-1.21	4.85	1.80
	0.8	-0.145	-0.488	1.953	-1.01	1.85	2.99
	1.0	-1.260	0.705	-6.975	0.00	0.00	-0.00
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	1.477	-0.481	-0.011	2.82	0.53	-0.09
	0.4	0.614	0.286	-0.411	1.77	2.21	-0.04
	0.6	-1.196	1.115	-0.054	-0.84	2.98	1.11
	0.8	-0.097	-0.398	1.538	-0.58	1.01	2.05
	1.0	-1.030	0.628	-5.490	0.00	0.00	-0.00
0.6	0.	-0.	0.	0.	0.	-0.	0.
	0.2	0.880	-0.292	-0.000	-0.04	-0.20	-0.00
	0.4	0.359	0.171	-0.250	-0.14	-0.22	-0.14
	0.6	-0.718	0.666	-0.043	-0.09	-0.26	-0.09
	0.8	-0.041	-0.256	0.936	0.12	-0.32	0.18
	1.0	-0.664	0.454	-3.337	-0.00	-0.00	-0.00
0.8	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	0.289	-0.099	0.003	-4.59	-1.06	0.14
	0.4	0.115	0.056	-0.084	-3.00	-3.79	-0.08
	0.6	-0.238	0.219	-0.019	1.26	-5.07	-1.89
	0.8	-0.006	-0.093	0.316	1.07	-1.95	-3.11
	1.0	-0.242	0.186	-1.125	-0.00	-0.00	0.00
1.0	0.	0.	0.	0.	0.	-0.	-0.
	0.2	0.000	0.000	0.000	-11.46	-1.83	0.33
	0.4	-0.000	-0.000	-0.000	-6.98	-8.69	0.41
	0.6	-0.000	-0.000	-0.000	3.55	-11.74	-4.36
	0.8	0.000	0.000	0.000	2.13	-3.55	-8.66
	1.0	-0.000	-0.000	-0.000	-0.00	-0.00	0.00

TABLE V. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.30

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	3.46	-3.72	0.48	13.98	1.06	0.05
	0.2	2.03	-1.48	-0.57	11.34	3.61	0.06
	0.4	-0.94	2.87	-1.57	5.52	8.24	1.01
	0.6	-2.45	4.03	1.19	0.87	8.94	3.34
	0.8	-0.71	-1.04	4.54	-0.27	3.92	3.48
	1.0	0.10	-4.25	-15.52	0.00	0.00	-0.00
0.2	0.	3.21	-3.46	0.47	12.45	0.75	0.08
	0.2	1.88	-1.38	-0.53	10.06	3.12	0.01
	0.4	-0.88	2.68	-1.49	4.81	7.39	0.80
	0.6	-2.28	3.73	1.10	0.67	7.99	3.05
	0.8	-0.64	-1.00	4.27	-0.27	3.42	3.31
	1.0	0.08	-3.82	-14.56	0.00	0.00	-0.00
0.4	0.	2.51	-2.74	0.42	7.77	-0.02	0.16
	0.2	1.46	-1.08	-0.41	6.17	1.70	-0.09
	0.4	-0.70	2.13	-1.23	2.72	4.74	0.24
	0.6	-1.78	2.91	0.85	0.16	5.05	2.09
	0.8	-0.47	-0.87	3.47	-0.24	1.93	2.62
	1.0	0.04	-2.68	-11.75	0.00	0.00	-0.00
0.6	0.	1.50	-1.68	0.31	-0.48	-0.85	0.21
	0.2	0.87	-0.65	-0.24	-0.56	-0.51	-0.15
	0.4	-0.44	1.32	-0.82	-0.64	-0.04	-0.49
	0.6	-1.07	1.75	0.49	-0.44	-0.23	0.18
	0.8	-0.26	-0.61	2.22	-0.08	-0.52	0.81
	1.0	-0.01	-1.30	-7.44	-0.00	0.00	-0.00
0.8	0.	0.50	-0.58	0.13	-13.00	-0.84	-0.03
	0.2	0.28	-0.22	-0.08	-10.52	-3.27	-0.02
	0.4	-0.15	0.46	-0.31	-5.05	-7.67	-0.91
	0.6	-0.36	0.58	0.16	-0.71	-8.37	-3.18
	0.8	-0.07	-0.25	0.80	0.31	-3.67	-3.38
	1.0	-0.02	-0.28	-2.65	-0.00	-0.00	0.00
1.0	0.	-0.00	-0.00	0.00	-31.00	1.61	-1.21
	0.2	-0.00	-0.00	0.00	-24.30	-6.18	0.60
	0.4	-0.00	-0.00	-0.00	-10.05	-19.52	0.06
	0.6	0.00	0.00	-0.00	0.00	-20.16	-8.77
	0.8	0.00	0.00	0.00	0.97	-6.65	-12.27
	1.0	-0.00	-0.00	-0.00	-0.00	-0.00	0.00

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TABLE VI. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.30

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	-0.	0.	0.	-0.	0.	0.
	0.2	6.56	-1.40	-0.40	7.06	3.51	0.10
	0.4	3.24	1.75	-1.07	5.36	8.17	1.01
	0.6	-4.81	5.20	0.62	-0.94	9.83	2.94
	0.8	-1.47	-0.70	4.38	-1.66	4.57	3.19
	1.0	-1.69	-3.25	-16.11	0.00	0.00	-0.00
0.2	0.	-0.	0.	0.	-0.	0.	0.
	0.2	6.11	-1.33	-0.36	6.48	3.04	0.06
	0.4	3.01	1.62	-1.01	4.84	7.27	0.84
	0.6	-4.49	4.84	0.56	-0.96	8.81	2.67
	0.8	-1.34	-0.69	4.12	-1.52	4.01	3.04
	1.0	-1.65	-2.83	-15.15	0.00	0.00	-0.00
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	4.83	-1.09	-0.27	4.53	1.67	-0.03
	0.4	2.34	1.26	-0.83	3.19	4.53	0.36
	0.6	-3.58	3.81	0.39	-0.92	5.63	1.78
	0.8	-0.99	-0.64	3.36	-1.03	2.33	2.42
	1.0	-1.51	-1.78	-12.31	0.00	0.00	-0.00
0.6	0.	-0.	0.	0.	0.	-0.	0.
	0.2	2.96	-0.71	-0.14	0.57	-0.51	-0.10
	0.4	1.40	0.75	-0.54	0.07	-0.28	-0.33
	0.6	-2.22	2.33	0.19	-0.51	-0.13	0.07
	0.8	-0.53	-0.49	2.16	-0.03	-0.51	0.77
	1.0	-1.14	-0.62	-7.88	0.00	-0.00	-0.00
0.8	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	1.01	-0.26	-0.04	-6.71	-3.22	-0.06
	0.4	0.46	0.25	-0.20	-5.07	-7.58	-0.92
	0.6	-0.77	0.79	0.04	0.96	-9.20	-2.81
	0.8	-0.14	-0.22	0.79	1.65	-4.31	-3.08
	1.0	-0.50	0.02	-2.85	-0.00	-0.00	0.00
1.0	0.	0.	-0.	0.	0.	-0.	-0.
	0.2	0.00	0.00	0.00	-19.59	-5.87	0.21
	0.4	0.00	-0.00	0.00	-13.05	-18.14	-0.76
	0.6	-0.00	-0.00	0.00	4.70	-22.78	-7.29
	0.8	-0.00	0.00	0.00	3.92	-8.16	-11.46
	1.0	-0.00	-0.00	-0.00	-0.00	-0.00	0.00

TABLE VII. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.40

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	7.93	-7.00	-0.20	21.23	5.14	0.27
	0.2	4.96	-2.68	-1.09	17.95	7.57	0.70
	0.4	-1.35	5.88	-1.43	10.42	11.83	2.00
	0.6	-5.09	8.53	2.52	3.59	11.81	3.40
	0.8	-2.48	-1.66	5.83	0.55	5.34	2.62
	1.0	-1.05	-13.37	-21.83	0.00	0.00	0.00
0.2	0.	7.37	-6.57	-0.13	19.02	4.23	0.24
	0.2	4.61	-2.51	-1.02	16.00	6.56	0.56
	0.4	-1.28	5.51	-1.40	9.12	10.66	1.71
	0.6	-4.75	7.94	2.34	2.97	10.70	3.19
	0.8	-2.28	-1.62	5.56	0.39	4.76	2.62
	1.0	-0.93	-12.14	-20.72	0.00	0.00	0.00
0.4	0.	5.81	-5.31	0.02	12.14	1.69	0.16
	0.2	3.61	-2.02	-0.81	10.00	3.62	0.17
	0.4	-1.07	4.45	-1.28	5.23	6.99	0.83
	0.6	-3.78	6.28	1.84	1.25	7.10	2.41
	0.8	-1.74	-1.47	4.73	-0.03	2.91	2.45
	1.0	-0.62	-8.82	-17.32	0.00	0.00	-0.00
0.6	0.	3.54	-3.38	0.14	-0.26	-1.80	0.09
	0.2	2.17	-1.26	-0.50	-0.58	-1.02	-0.32
	0.4	-0.71	2.83	-0.97	-1.11	0.23	-0.55
	0.6	-2.34	3.85	1.10	-1.11	0.26	0.57
	0.8	-0.99	-1.13	3.24	-0.43	-0.44	1.36
	1.0	-0.29	-4.53	-11.61	-0.00	0.00	-0.00
0.8	0.	1.20	-1.22	0.12	-19.81	-4.62	-0.13
	0.2	0.73	-0.44	-0.17	-16.71	-6.89	-0.61
	0.4	-0.27	1.03	-0.43	-9.59	-10.96	-1.95
	0.6	-0.81	1.31	0.36	-3.15	-11.19	-3.29
	0.8	-0.30	-0.51	1.28	-0.38	-5.23	-2.48
	1.0	-0.07	-1.08	-4.46	-0.00	-0.00	-0.00
1.0	0.	0.00	0.00	0.00	-49.39	-3.24	-1.37
	0.2	0.00	0.00	0.00	-39.98	-13.12	-0.12
	0.4	-0.00	-0.00	-0.00	-19.37	-29.69	-1.73
	0.6	-0.00	-0.00	-0.00	-3.24	-29.23	-10.84
	0.8	0.00	0.00	0.00	0.62	-9.89	-13.27
	1.0	-0.00	-0.00	-0.00	-0.00	-0.00	0.00

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TABLE VIII. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.40

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	-0.	0.	0.	-0.	0.	0.
	0.2	12.19	-0.88	-0.73	7.41	6.27	0.63
	0.4	6.74	5.13	-0.98	6.47	11.95	1.85
	0.6	-7.91	10.65	1.89	0.51	12.85	3.15
	0.8	-3.43	-1.08	5.64	-1.09	6.02	2.46
	1.0	-2.36	-12.67	-22.17	0.00	0.00	0.00
0.2	0.	-0.	0.	0.	-0.	0.	0.
	0.2	11.43	-0.89	-0.68	6.99	5.52	0.52
	0.4	6.28	4.76	-0.96	5.96	10.75	1.60
	0.6	-7.46	9.95	1.73	0.25	11.68	2.93
	0.8	-3.17	-1.09	5.39	-1.11	5.40	2.45
	1.0	-2.31	-11.39	-21.10	0.00	0.00	0.00
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	9.22	-0.88	-0.53	5.39	3.24	0.20
	0.4	4.98	3.71	-0.87	4.23	6.96	0.86
	0.6	-6.12	7.94	1.29	-0.41	7.86	2.15
	0.8	-2.43	-1.07	4.59	-1.04	3.41	2.27
	1.0	-2.14	-7.97	-17.77	0.00	0.00	-0.00
0.6	0.	-0.	0.	0.	-0.	-0.	0.
	0.2	5.83	-0.72	-0.30	1.52	-0.61	-0.22
	0.4	3.06	2.23	-0.65	0.60	0.02	-0.36
	0.6	-3.99	4.95	0.70	-1.00	0.48	0.40
	0.8	-1.39	-0.90	3.16	-0.46	-0.33	1.26
	1.0	-1.71	-3.71	-12.06	0.00	-0.00	-0.00
0.8	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	2.08	-0.34	-0.09	-7.07	-5.89	-0.54
	0.4	1.04	0.75	-0.28	-6.20	-11.17	-1.79
	0.6	-1.48	1.74	0.19	-0.40	-12.16	-3.06
	0.8	-0.41	-0.45	1.26	1.26	-5.94	-2.29
	1.0	-0.84	-0.61	-4.72	-0.00	-0.00	-0.00
1.0	0.	-0.	0.	-0.	0.	-0.	-0.
	0.2	0.00	0.00	-0.00	-25.32	-11.73	-0.47
	0.4	0.00	0.00	0.00	-18.26	-28.67	-2.40
	0.6	-0.00	-0.00	0.00	3.86	-32.93	-9.34
	0.8	-0.00	0.00	-0.00	4.44	-11.89	-12.48
	1.0	-0.00	-0.00	0.00	-0.00	-0.00	0.00

TABLE IX. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.50

x/β	y	$\theta_1 \times 10^3$	$\theta_2 \times 10^3$	$\theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	14.04	-9.27	-0.94	28.30	10.71	1.00
	0.2	9.35	-2.98	-1.44	24.71	12.39	1.48
	0.4	-0.86	9.48	-0.85	16.13	14.93	2.55
	0.6	-7.76	13.05	3.78	7.40	13.24	3.09
	0.8	-5.72	-3.23	6.18	1.95	5.76	1.86
	1.0	-6.04	-24.97	-25.93	0.00	0.00	0.00
0.2	0.	13.10	-8.79	-0.81	25.48	9.09	0.83
	0.2	8.70	-2.83	-1.36	22.13	10.82	1.24
	0.4	-0.86	8.92	-0.90	14.18	13.51	2.26
	0.6	-7.30	12.23	3.54	6.24	12.16	2.98
	0.8	-5.31	-3.11	6.00	1.55	5.27	1.95
	1.0	-5.37	-22.85	-24.90	0.00	0.00	0.00
0.4	0.	10.41	-7.32	-0.49	16.60	4.39	0.39
	0.2	6.87	-2.39	-1.11	14.10	6.13	0.55
	0.4	-0.82	7.29	-1.00	8.32	9.02	1.31
	0.6	-5.95	9.84	2.84	2.96	8.51	2.47
	0.8	-4.14	-2.75	5.36	0.46	3.54	2.10
	1.0	-3.64	-16.97	-21.53	0.00	0.00	0.00
0.6	0.	6.43	-4.90	-0.10	0.25	-2.64	-0.14
	0.2	4.20	-1.61	-0.72	-0.34	-1.45	-0.45
	0.4	-0.65	4.74	-0.94	-1.47	0.61	-0.41
	0.6	-3.82	6.20	1.77	-1.85	1.01	0.95
	0.8	-2.48	-2.07	3.98	-0.91	-0.06	1.66
	1.0	-1.65	-9.01	-15.26	-0.00	0.00	-0.00
0.8	0.	2.23	-1.89	0.08	-26.45	-9.94	-0.69
	0.2	1.43	-0.62	-0.26	-23.05	-11.39	-1.36
	0.4	-0.29	1.79	-0.51	-14.94	-13.76	-2.64
	0.6	-1.39	2.20	0.61	-6.70	-12.69	-2.98
	0.8	-0.81	-0.97	1.74	-1.67	-5.98	-1.52
	1.0	-0.32	-2.20	-6.33	-0.00	-0.00	-0.00
1.0	0.	0.00	-0.00	-0.00	-68.87	-11.78	-2.26
	0.2	0.00	-0.00	0.00	-57.29	-22.42	-1.28
	0.4	-0.00	0.00	0.00	-31.11	-39.47	-3.28
	0.6	-0.00	0.00	0.00	-8.49	-36.23	-12.10
	0.8	0.00	-0.00	-0.00	-0.28	-11.87	-13.64
	1.0	-0.00	0.00	0.00	-0.00	-0.00	0.00

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TABLE X. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.50

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	-0.	0.	0.	-0.	0.	0.
	0.2	17.38	1.53	-0.68	6.70	8.14	1.07
	0.4	10.51	10.62	-0.33	6.62	14.03	2.28
	0.6	-9.75	16.75	3.23	1.91	13.86	2.92
	0.8	-5.31	-2.26	6.04	-0.21	6.26	1.79
	1.0	-4.48	-24.78	-26.08	0.00	0.00	0.00
0.2	0.	-0.	0.	0.	-0.	0.	0.
	0.2	16.41	1.29	-0.65	6.51	7.31	0.91
	0.4	9.87	9.86	-0.39	6.23	12.81	2.04
	0.6	-9.31	15.71	2.99	1.49	12.83	2.80
	0.8	-4.97	-2.21	5.87	-0.39	5.78	1.86
	1.0	-4.23	-22.56	-25.10	0.00	0.00	0.00
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	13.53	0.67	-0.55	5.55	4.63	0.45
	0.4	7.97	7.73	-0.51	4.78	8.79	1.25
	0.6	-7.92	12.70	2.31	0.30	9.24	2.27
	0.8	-3.95	-2.04	5.24	-0.79	4.05	1.97
	1.0	-3.58	-16.43	-21.85	0.00	0.00	0.00
0.6	0.	-0.	0.	0.	-0.	-0.	0.
	0.2	8.88	0.03	-0.35	2.45	-0.29	-0.26
	0.4	5.04	4.68	-0.54	1.28	0.83	-0.22
	0.6	-5.46	8.09	1.33	-1.26	1.53	0.77
	0.8	-2.40	-1.66	3.89	-0.88	0.21	1.53
	1.0	-2.61	-8.23	-15.70	0.00	0.00	-0.00
0.8	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	3.34	-0.20	-0.11	-6.28	-7.84	-0.98
	0.4	1.79	1.59	-0.31	-6.37	-13.26	-2.31
	0.6	-2.19	2.95	0.39	-1.88	-13.29	-2.87
	0.8	-0.76	-0.85	1.71	0.44	-6.55	-1.43
	1.0	-1.27	-1.60	-6.66	-0.00	-0.00	-0.00
1.0	0.	0.	0.	0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-28.82	-17.40	-1.29
	0.4	0.00	-0.00	-0.00	-22.04	-37.67	-3.82
	0.6	-0.00	0.00	0.00	2.28	-40.43	-10.63
	0.8	0.00	-0.00	0.00	4.27	-14.16	-12.89
	1.0	-0.00	0.00	-0.00	-0.00	-0.00	0.00

TABLE XI. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 0.75

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	35.86	-5.42	-1.20	41.3	22.6	2.8
	0.2	26.89	3.18	-1.03	37.5	22.1	2.9
	0.4	6.00	19.16	0.84	27.7	19.6	2.9
	0.6	-13.05	19.47	5.51	15.5	13.7	2.2
	0.8	-21.74	-12.91	5.09	4.9	5.1	0.9
	1.0	-38.24	-56.96	-31.96	0.0	0.0	0.0
0.2	0.	33.66	-5.62	-1.13	37.7	19.9	2.4
	0.2	25.15	2.67	-1.04	34.1	19.7	2.6
	0.4	5.36	18.12	0.65	24.8	18.0	2.7
	0.6	-12.56	18.59	5.28	13.6	12.9	2.3
	0.8	-20.27	-12.07	5.18	4.2	4.9	1.0
	1.0	-34.53	-53.04	-31.20	0.0	0.0	0.0
0.4	0.	27.25	-5.88	-0.87	26.1	11.7	1.3
	0.2	20.16	1.42	-1.04	23.1	12.3	1.5
	0.4	3.70	15.06	0.12	15.7	12.7	2.0
	0.6	-10.93	15.80	4.53	7.7	10.1	2.2
	0.8	-16.15	-9.80	5.28	2.1	4.1	1.3
	1.0	-24.49	-41.49	-28.53	0.0	0.0	0.0
0.6	0.	17.39	-5.27	-0.39	3.0	-2.6	-0.5
	0.2	12.67	0.13	-0.86	1.8	-1.0	-0.4
	0.4	1.73	10.20	-0.51	-0.8	2.0	0.2
	0.6	-7.81	10.86	3.14	-2.5	3.0	1.5
	0.8	-10.15	-6.65	4.81	-1.5	1.2	1.6
	1.0	-11.65	-23.85	-22.55	0.0	0.0	-0.0
0.8	0.	6.30	-2.80	0.06	-38.7	-22.1	-2.4
	0.2	4.51	-0.33	-0.40	-35.2	-21.0	-2.9
	0.4	0.35	4.18	-0.63	-26.2	-18.0	-3.3
	0.6	-3.26	4.33	1.23	-15.2	-13.2	-1.8
	0.8	-3.68	-2.99	2.74	-5.2	-5.9	0.2
	1.0	-2.02	-6.21	-11.13	-0.0	-0.0	-0.0
1.0	0.	-0.00	-0.00	0.00	-114.4	-37.4	-5.5
	0.2	-0.00	-0.00	0.00	-98.7	-47.0	-4.4
	0.4	0.00	0.00	-0.00	-60.9	-59.4	-6.0
	0.6	0.00	0.00	-0.00	-22.5	-47.1	-13.7
	0.8	-0.00	-0.00	0.00	-2.4	-13.8	-13.9
	1.0	0.00	0.00	-0.00	-0.0	-0.0	0.0

TABLE XII. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 0.75

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	25.99	10.88	0.41	3.92	7.53	1.13
	0.4	18.21	26.09	1.94	4.70	11.72	1.88
	0.6	-10.19	29.23	5.62	2.70	10.29	1.78
	0.8	-8.65	-7.13	5.58	0.76	4.20	0.78
	1.0	-11.83	-50.93	-31.18	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	24.99	9.94	0.31	4.13	7.23	1.06
	0.4	17.34	24.44	1.70	4.74	11.45	1.84
	0.6	-10.10	27.77	5.33	2.44	10.26	1.85
	0.8	-8.29	-6.71	5.61	0.57	4.28	0.88
	1.0	-10.97	-47.68	-30.53	0.00	0.00	0.00
0.4	0.	-0.	0.	0.	-0.	0.	0.
	0.2	21.74	7.32	0.07	4.51	5.83	0.76
	0.4	14.66	19.66	1.04	4.53	9.79	1.55
	0.6	-9.54	23.31	4.42	1.43	9.37	1.94
	0.8	-7.11	-5.59	5.56	-0.07	4.10	1.20
	1.0	-8.60	-37.77	-28.19	0.00	0.00	0.00
0.6	0.	-0.	0.	0.	-0.	0.	0.
	0.2	15.65	3.77	-0.13	3.81	1.72	0.01
	0.4	10.05	12.43	0.21	2.79	4.03	0.46
	0.6	-7.74	15.84	2.86	-0.74	4.74	1.44
	0.8	-4.90	-4.07	4.92	-1.07	2.03	1.56
	1.0	-5.44	-21.84	-22.68	0.00	0.00	-0.00
0.8	0.	-0.	0.	0.	-0.	-0.	0.
	0.2	6.72	0.79	-0.10	-2.72	-7.81	-1.26
	0.4	3.99	4.50	-0.24	-4.22	-11.45	-2.19
	0.6	-3.82	6.34	0.97	-3.44	-10.08	-1.55
	0.8	-1.86	-2.11	2.76	-1.04	-5.08	0.21
	1.0	-2.37	-5.27	-11.53	-0.00	-0.00	-0.00
1.0	0.	0.	0.	-0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-32.18	-24.83	-2.39
	0.4	0.00	-0.00	-0.00	-26.19	-48.48	-5.47
	0.6	-0.00	0.00	-0.00	-0.07	-48.33	-11.86
	0.8	0.00	0.00	0.00	3.86	-16.00	-13.17
	1.0	-0.00	-0.00	0.00	-0.00	-0.00	0.00

TABLE XIII. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 1.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	62.71	7.53	0.22	42.3	25.0	3.2
	0.2	49.60	15.43	0.45	38.8	23.5	3.1
	0.4	17.34	28.12	2.17	29.4	18.9	2.6
	0.6	-17.71	20.00	5.83	16.9	11.7	1.6
	0.8	-44.59	-27.10	3.20	5.4	3.9	0.6
	1.0	-82.21	-87.09	-36.04	0.0	0.0	0.0
0.2	0.	59.14	6.09	0.09	39.6	22.9	2.9
	0.2	46.58	13.93	0.28	36.2	21.7	2.8
	0.4	15.80	26.73	1.95	27.2	17.9	2.5
	0.6	-17.23	19.54	5.71	15.5	11.4	1.7
	0.8	-41.67	-25.27	3.45	4.9	3.9	0.6
	1.0	-75.42	-82.11	-35.34	0.0	0.0	0.0
0.4	0.	48.62	2.41	-0.16	30.0	15.7	1.9
	0.2	37.81	9.89	-0.14	27.0	15.5	2.0
	0.4	11.60	22.55	1.26	19.4	14.0	2.1
	0.6	-15.59	17.75	5.23	10.4	9.9	1.7
	0.8	-33.39	-20.19	4.11	3.2	3.8	0.8
	1.0	-56.14	-66.89	-32.98	0.0	0.0	0.0
0.6	0.	31.98	-1.52	-0.20	7.9	0.5	-0.2
	0.2	24.39	4.74	-0.53	6.1	1.8	0.1
	0.4	6.12	15.71	0.18	2.3	4.2	0.8
	0.6	-12.03	13.53	4.04	-0.6	4.7	1.5
	0.8	-21.32	-13.14	4.67	-0.8	2.1	1.2
	1.0	-28.94	-41.50	-27.68	0.0	0.0	0.0
0.8	0.	12.17	-2.55	0.12	-39.6	-25.8	-3.1
	0.2	9.08	0.85	-0.43	-36.8	-23.4	-3.3
	0.4	1.63	6.84	-0.61	-29.1	-17.5	-2.9
	0.6	-5.66	6.15	1.84	-18.3	-10.9	-0.8
	0.8	-8.13	-5.72	3.48	-6.8	-4.6	1.0
	1.0	-5.43	-11.97	-15.65	-0.0	-0.0	-0.0
1.0	0.	0.00	-0.00	0.00	-142.7	-54.3	-7.6
	0.2	0.00	-0.00	-0.00	-124.4	-62.3	-6.4
	0.4	-0.00	0.00	-0.00	-79.0	-70.4	-7.4
	0.6	-0.00	0.00	0.00	-30.5	-52.0	-14.3
	0.8	0.00	-0.00	0.00	-3.0	-14.2	-14.0
	1.0	-0.00	0.00	-0.00	-0.0	-0.0	0.0

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TABLE XIV. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 1.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	29.83	17.63	1.37	1.89	4.37	0.67
	0.4	22.27	35.86	3.42	2.47	6.52	1.01
	0.6	-9.28	35.44	6.66	1.68	5.37	0.85
	0.8	-10.35	-10.97	4.99	0.56	2.03	0.32
	1.0	-16.29	-63.75	-33.10	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	29.09	16.47	1.21	2.21	4.70	0.71
	0.4	21.51	34.10	3.15	2.77	7.13	1.12
	0.6	-9.42	34.20	6.44	1.75	6.04	1.00
	0.8	-10.05	-10.34	5.09	0.54	2.37	0.41
	1.0	-15.39	-61.12	-32.70	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	26.46	12.98	0.75	3.11	5.13	0.74
	0.4	18.98	28.60	2.32	3.47	8.20	1.32
	0.6	-9.62	30.05	5.69	1.64	7.45	1.38
	0.8	-9.02	-8.55	5.32	0.32	3.16	0.69
	1.0	-12.61	-52.17	-31.17	0.00	0.00	0.00
0.6	0.	-0.	0.	0.	-0.	0.	0.
	0.2	20.66	7.55	0.19	3.96	3.55	0.37
	0.4	14.02	19.25	1.02	3.48	6.54	1.00
	0.6	-8.93	21.99	4.10	0.32	6.81	1.63
	0.8	-6.83	-6.00	5.32	-0.57	3.11	1.25
	1.0	-8.14	-34.49	-27.02	0.00	0.00	0.00
0.8	0.	-0.	0.	0.	-0.	-0.	0.
	0.2	10.07	1.98	-0.08	0.35	-4.79	-0.94
	0.4	6.25	7.53	-0.11	-1.39	-6.07	-1.31
	0.6	-5.32	9.76	1.61	-3.04	-4.51	-0.23
	0.8	-3.01	-3.14	3.66	-1.50	-2.38	1.09
	1.0	-3.43	-10.32	-15.98	-0.00	-0.00	-0.00
1.0	0.	-0.	0.	-0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-32.73	-26.26	-2.60
	0.4	0.00	-0.00	-0.00	-26.90	-50.40	-5.74
	0.6	-0.00	0.00	0.00	-0.48	-49.54	-12.03
	0.8	-0.00	-0.00	-0.00	3.82	-16.15	-13.19
	1.0	0.00	0.00	0.00	-0.00	-0.00	0.00

TABLE XV. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 1.25

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	86.0	20.6	1.8	34.4	20.6	2.6
	0.2	69.4	26.9	1.9	31.5	19.1	2.5
	0.4	27.3	34.7	3.1	23.9	14.8	1.9
	0.6	-22.0	18.3	5.7	13.6	8.7	1.1
	0.8	-65.0	-39.6	1.6	4.2	2.7	0.4
	1.0	-117.7	-109.1	-38.8	0.0	0.0	0.0
0.2	0.	81.5	18.2	1.5	33.6	20.0	2.6
	0.2	65.5	24.7	1.6	30.8	18.6	2.4
	0.4	25.2	33.2	2.8	23.3	14.7	1.9
	0.6	-21.4	18.3	5.6	13.3	8.8	1.2
	0.8	-61.1	-37.1	1.9	4.2	2.9	0.4
	1.0	-109.6	-103.9	-38.2	0.0	0.0	0.0
0.4	0.	68.1	11.7	0.8	29.1	16.3	2.0
	0.2	54.1	18.5	0.9	26.4	15.6	2.0
	0.4	19.2	28.6	2.2	19.5	13.2	1.9
	0.6	-19.5	17.8	5.5	10.9	8.7	1.4
	0.8	-49.6	-30.0	2.9	3.4	3.1	0.5
	1.0	-85.6	-87.6	-36.0	0.0	0.0	0.0
0.6	0.	46.2	3.4	0.2	13.2	4.6	0.4
	0.2	35.9	9.9	-0.0	11.2	5.4	0.6
	0.4	10.7	20.6	0.9	6.7	6.6	1.2
	0.6	-15.7	15.1	4.6	2.5	6.0	1.5
	0.8	-32.3	-19.5	4.1	0.5	2.6	0.9
	1.0	-48.1	-58.3	-31.2	0.0	0.0	0.0
0.8	0.	18.6	-1.8	0.2	-32.0	-22.9	-2.9
	0.2	14.1	2.3	-0.4	-30.2	-20.1	-2.9
	0.4	3.0	9.5	-0.5	-25.1	-13.6	-2.1
	0.6	-8.1	7.9	2.4	-16.7	-7.3	0.0
	0.8	-12.8	-8.4	4.0	-6.4	-2.8	1.3
	1.0	-10.4	-19.1	-19.6	-0.0	-0.0	-0.0
1.0	0.	0.0	-0.0	0.0	-153.8	-60.8	-8.4
	0.2	0.0	-0.0	0.0	-134.3	-68.2	-7.1
	0.4	0.0	0.0	-0.0	-85.7	-74.3	-7.9
	0.6	-0.0	0.0	-0.0	-33.0	-53.5	-14.5
	0.8	-0.0	-0.0	0.0	-3.0	-14.2	-14.0
	1.0	0.0	0.0	-0.0	-0.0	-0.0	0.0

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TABLE XVI. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 1.25

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	31.36	20.94	1.85	0.82	2.07	0.31
	0.4	24.01	40.34	4.09	1.11	3.00	0.45
	0.6	-8.73	37.84	7.03	0.79	2.36	0.36
	0.8	-11.12	-12.95	4.68	0.27	0.84	0.13
	1.0	-18.05	-68.52	-33.77	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	30.88	19.97	1.71	1.11	2.61	0.40
	0.4	23.47	38.98	3.88	1.46	3.87	0.60
	0.6	-8.90	37.04	6.90	1.00	3.14	0.49
	0.8	-10.90	-12.39	4.77	0.33	1.17	0.18
	1.0	-17.45	-66.89	-33.54	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	29.02	16.74	1.25	2.04	3.97	0.60
	0.4	21.50	34.26	3.17	2.46	6.15	0.98
	0.6	-9.35	33.97	6.38	1.43	5.36	0.92
	0.8	-10.09	-10.58	5.05	0.40	2.17	0.40
	1.0	-15.23	-60.50	-32.58	0.00	0.00	0.00
0.6	0.	0.	0.	0.	-0.	0.	0.
	0.2	24.19	10.76	0.51	3.51	4.44	0.58
	0.4	16.99	24.76	1.76	3.50	7.52	1.21
	0.6	-9.45	26.69	5.03	1.07	7.31	1.52
	0.8	-8.18	-7.54	5.37	-0.07	3.28	0.91
	1.0	-10.61	-44.73	-29.63	0.00	0.00	0.00
0.8	0.	-0.	0.	0.	-0.	-0.	0.
	0.2	13.20	3.27	-0.05	2.28	-1.84	-0.54
	0.4	8.42	10.55	0.10	0.73	-1.33	-0.47
	0.6	-6.61	13.10	2.27	-2.16	0.04	0.67
	0.8	-4.11	-3.97	4.35	-1.48	-0.12	1.43
	1.0	-4.53	-16.23	-19.73	0.00	-0.00	-0.00
1.0	0.	-0.	0.	0.	0.	-0.	-0.
	0.2	-0.00	-0.00	-0.00	-32.78	-26.40	-2.62
	0.4	-0.00	-0.00	-0.00	-26.96	-50.57	-5.77
	0.6	-0.00	-0.00	0.00	-0.51	-49.61	-12.04
	0.8	-0.00	0.00	0.00	3.82	-16.14	-13.18
	1.0	0.00	-0.00	-0.00	-0.00	-0.00	0.00

TABLE XVII. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 1.50

x/β	y	$\Theta_1\times10^3$	$\Theta_2\times10^3$	$\Theta_3\times10^3$	$\Phi_1\times10^3$	$\Phi_2\times10^3$	$\Phi_3\times10^3$
0.	0.	102.0	29.9	3.0	24.2	14.5	1.8
	0.2	82.9	34.9	2.9	22.2	13.4	1.7
	0.4	33.8	38.8	3.6	16.7	10.1	1.3
	0.6	-25.5	16.5	5.4	9.4	5.8	0.7
	0.8	-79.1	-48.1	0.5	2.9	1.8	0.2
	1.0	-139.8	-122.4	-40.5	0.0	0.0	0.0
0.2	0.	97.3	27.2	2.7	25.3	15.1	1.9
	0.2	78.9	32.5	2.6	23.2	14.0	1.8
	0.4	31.7	37.4	3.4	17.5	10.8	1.4
	0.6	-24.7	16.8	5.5	9.9	6.3	0.8
	0.8	-74.9	-45.5	0.8	3.0	2.0	0.3
	1.0	-132.1	-117.7	-39.9	0.0	0.0	0.0
0.4	0.	82.9	19.4	1.7	26.0	15.1	1.9
	0.2	66.5	25.5	1.7	23.8	14.2	1.9
	0.4	25.3	33.0	2.8	17.8	11.6	1.6
	0.6	-22.5	17.3	5.5	10.1	7.3	1.0
	0.8	-62.2	-37.7	1.9	3.2	2.5	0.4
	1.0	-108.0	-102.4	-37.9	0.0	0.0	0.0
0.6	0.	58.3	8.1	0.5	17.6	8.2	0.9
	0.2	45.8	14.6	0.4	15.6	8.5	1.1
	0.4	15.0	24.6	1.5	10.6	8.6	1.4
	0.6	-18.4	16.1	5.0	5.3	6.7	1.3
	0.8	-41.7	-25.2	3.5	1.6	2.8	0.7
	1.0	-66.0	-72.7	-33.7	0.0	0.0	0.0
0.8	0.	24.9	-1.0	0.1	-21.7	-17.4	-2.4
	0.2	18.9	3.8	-0.4	-21.0	-14.7	-2.2
	0.4	4.4	11.9	-0.3	-18.5	-8.8	-1.2
	0.6	-10.3	9.5	3.0	-13.0	-3.7	0.6
	0.8	-17.1	-10.9	4.3	-5.2	-1.1	1.3
	1.0	-16.7	-26.9	-22.9	-0.0	-0.0	-0.0
1.0	0.	0.0	0.0	0.0	-156.4	-62.3	-8.6
	0.2	0.0	0.0	0.0	-136.6	-69.5	-7.3
	0.4	-0.0	0.0	-0.0	-87.1	-75.1	-8.0
	0.6	-0.0	-0.0	-0.0	-33.4	-53.7	-14.5
	0.8	0.0	-0.0	0.0	-2.8	-14.1	-13.9
	1.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0

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TABLE XVIII. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; $\beta = 1.50$

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	31.93	22.31	2.05	0.32	0.85	0.13
	0.4	24.67	42.10	4.35	0.44	1.21	0.18
	0.6	-8.52	38.64	7.14	0.32	0.91	0.13
	0.8	-11.44	-13.81	4.55	0.10	0.31	0.04
	1.0	-18.61	-70.03	-33.98	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	31.66	21.67	1.96	0.53	1.34	0.20
	0.4	24.34	41.25	4.22	0.72	1.94	0.29
	0.6	-8.63	38.21	7.08	0.51	1.53	0.23
	0.8	-11.29	-13.42	4.61	0.17	0.54	0.08
	1.0	-18.30	-69.18	-33.86	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	30.44	19.20	1.60	1.30	2.84	0.43
	0.4	22.99	37.81	3.70	1.65	4.30	0.67
	0.6	-9.04	36.23	6.77	1.08	3.61	0.59
	0.8	-10.71	-11.96	4.84	0.34	1.40	0.23
	1.0	-16.84	-65.19	-33.29	0.00	0.00	0.00
0.6	0.	0.	0.	0.	0.	0.	0.
	0.2	26.67	13.53	0.83	2.88	4.52	0.65
	0.4	19.22	29.26	2.41	3.15	7.32	1.19
	0.6	-9.56	30.28	5.72	1.41	6.78	1.28
	0.8	-9.15	-8.90	5.27	0.24	2.93	0.65
	1.0	-12.76	-52.55	-31.22	0.00	0.00	0.00
0.8	0.	-0.	0.	0.	-0.	0.	0.
	0.2	16.04	4.69	0.01	3.31	0.46	-0.18
	0.4	10.48	13.59	0.37	2.10	2.17	0.18
	0.6	-7.63	16.30	2.93	-1.25	3.23	1.21
	0.8	-5.14	-4.71	4.83	-1.25	1.46	1.47
	1.0	-5.69	-22.49	-22.76	0.00	0.00	-0.00
1.0	0.	-0.	0.	-0.	0.	-0.	-0.
	0.2	-0.00	-0.00	0.00	-32.78	-26.40	-2.62
	0.4	0.00	0.00	0.00	-26.95	-50.56	-5.76
	0.6	0.00	0.00	-0.00	-0.50	-49.60	-12.04
	0.8	-0.00	-0.00	-0.00	3.82	-16.14	-13.18
	1.0	0.00	0.00	0.00	-0.00	-0.00	0.00

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TABLE XIX. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; $\beta = 1.75$

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	111.7	35.6	3.7	15.5	9.2	1.2
	0.2	91.0	39.7	3.5	14.1	8.5	1.1
	0.4	37.5	41.0	3.8	10.6	6.3	0.8
	0.6	-27.9	15.1	5.3	5.9	3.5	0.4
	0.8	-87.5	-53.1	-0.1	1.7	1.0	0.1
	1.0	-151.6	-129.3	-41.3	0.0	0.0	0.0
0.2	0.	107.5	33.1	3.4	17.7	10.6	1.3
	0.2	87.4	37.5	3.2	16.2	9.8	1.2
	0.4	35.7	39.9	3.7	12.2	7.4	0.9
	0.6	-27.1	15.5	5.3	6.8	4.2	0.5
	0.8	-83.7	-50.8	0.2	2.1	1.3	0.2
	1.0	-145.4	-125.6	-40.9	0.0	0.0	0.0
0.4	0.	93.7	25.3	2.4	22.3	13.2	1.7
	0.2	75.7	30.7	2.3	20.5	12.4	1.6
	0.4	29.7	36.0	3.2	15.5	9.8	1.3
	0.6	-24.6	16.6	5.4	8.8	5.9	0.8
	0.8	-71.5	-43.4	1.1	2.8	1.9	0.3
	1.0	-124.1	-112.6	-39.2	0.0	0.0	0.0
0.6	0.	68.4	12.4	1.0	20.6	10.9	1.3
	0.2	54.2	18.8	0.9	18.5	10.7	1.4
	0.4	18.8	27.9	2.0	13.4	9.8	1.5
	0.6	-20.4	16.7	5.3	7.3	7.0	1.2
	0.8	-49.8	-30.1	2.9	2.3	2.7	0.5
	1.0	-82.0	-84.6	-35.5	0.0	0.0	0.0
0.8	0.	30.9	0.1	0.1	-11.9	-11.6	-1.7
	0.2	23.6	5.4	-0.3	-12.1	-9.3	-1.4
	0.4	5.9	14.3	-0.0	-11.8	-4.4	-0.4
	0.6	-12.2	11.1	3.5	-9.0	-0.6	1.0
	0.8	-21.3	-13.2	4.4	-3.7	0.2	1.3
	1.0	-23.9	-35.0	-25.6	-0.0	-0.0	-0.0
1.0	0.	-0.0	-0.0	0.0	-156.6	-62.4	-8.6
	0.2	-0.0	-0.0	0.0	-136.7	-69.5	-7.3
	0.4	0.0	0.0	0.0	-87.1	-75.1	-8.0
	0.6	0.0	0.0	-0.0	-33.3	-53.6	-14.5
	0.8	-0.0	-0.0	-0.0	-2.8	-14.1	-13.9
	1.0	0.0	0.0	0.0	-0.0	-0.0	-0.0

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TABLE XX. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 1.75

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	32.13	22.82	2.12	0.12	0.31	0.04
	0.4	24.89	42.71	4.43	0.16	0.43	0.06
	0.6	-8.46	38.86	7.17	0.11	0.31	0.04
	0.8	-11.56	-14.14	4.51	0.03	0.10	0.01
	1.0	-18.76	-70.41	-34.03	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	31.99	22.46	2.07	0.25	0.65	0.10
	0.4	24.72	42.26	4.37	0.34	0.92	0.14
	0.6	-8.51	38.66	7.14	0.24	0.70	0.10
	0.8	-11.48	-13.91	4.54	0.08	0.24	0.03
	1.0	-18.62	-70.05	-33.98	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	31.24	20.77	1.82	0.81	1.91	0.29
	0.4	23.86	40.01	4.03	1.07	2.83	0.44
	0.6	-8.79	37.51	6.97	0.74	2.30	0.36
	0.8	-11.09	-12.89	4.70	0.24	0.86	0.13
	1.0	-17.78	-67.78	-33.66	0.00	0.00	0.00
0.6	0.	0.	0.	0.	0.	0.	0.
	0.2	28.40	15.85	1.13	2.26	4.11	0.61
	0.4	20.88	32.87	2.95	2.64	6.47	1.04
	0.6	-9.44	32.94	6.20	1.43	5.76	1.02
	0.8	-9.84	-10.11	5.12	0.37	2.40	0.46
	1.0	-14.51	-58.28	-32.22	0.00	0.00	0.00
0.8	0.	-0.	0.	0.	-0.	0.	0.
	0.2	18.55	6.20	0.10	3.76	2.12	0.11
	0.4	12.38	16.59	0.70	2.91	4.57	0.64
	0.6	-8.39	19.30	3.54	-0.45	5.29	1.48
	0.8	-6.06	-5.43	5.13	-0.93	2.46	1.38
	1.0	-6.91	-28.69	-25.15	0.00	0.00	-0.00
1.0	0.	0.	-0.	-0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-32.78	-26.40	-2.62
	0.4	-0.00	-0.00	0.00	-26.95	-50.55	-5.76
	0.6	-0.00	-0.00	0.00	-0.50	-49.60	-12.03
	0.8	0.00	0.00	-0.00	3.82	-16.13	-13.18
	1.0	-0.00	-0.00	0.00	-0.00	-0.00	0.00

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TABLE XXI. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 2.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	117.2	38.8	4.1	9.1	5.4	0.7
	0.2	95.5	42.3	3.8	8.3	4.9	0.6
	0.4	39.4	42.1	4.0	6.1	3.6	0.5
	0.6	-29.5	14.1	5.1	3.4	2.0	0.2
	0.8	-92.1	-55.8	-0.4	1.0	0.6	0.1
	1.0	-157.2	-132.5	-41.7	0.0	0.0	0.0
0.2	0.	113.7	36.8	3.8	11.9	7.1	0.9
	0.2	92.6	40.6	3.6	10.8	6.5	0.8
	0.4	38.0	41.3	3.9	8.1	4.9	0.6
	0.6	-28.7	14.6	5.2	4.5	2.7	0.3
	0.8	-89.0	-54.0	-0.2	1.3	0.8	0.1
	1.0	-152.7	-129.9	-41.4	0.0	0.0	0.0
0.4	0.	101.6	29.7	3.0	18.6	11.2	1.4
	0.2	82.3	34.5	2.8	17.1	10.3	1.3
	0.4	33.0	38.2	3.5	12.9	8.0	1 1
	0.6	-26.2	15.9	5.4	7.4	4.7	0.6
	0.8	-78.4	-47.5	0.6	2.3	1.5	0.2
	1.0	-135.6	-119.6	-40.1	0.0	0.0	0.0
0.6	0.	77.0	16.5	1.4	22.0	12.4	1.6
	0.2	61.4	22.7	1.4	20.1	11.9	1.6
	0.4	22.3	30.7	2.5	14.9	10.2	1.5
	0.6	-21.9	16.9	5.4	8.4	6.9	1.1
	0.8	-57.0	-34.4	2.3	2.7	2.5	0.4
	1.0	-96.1	-94.4	-36.8	0.0	0.0	0.0
0.8	0.	36.8	1.3	0.2	-3.7	-6.4	-1.0
	0.2	28.3	7.0	-0.2	-4.5	-4.6	-0.7
	0.4	7.5	16.5	0.3	-5.8	-0.6	0.2
	0.6	-13.8	12.5	3.9	-5.3	1.8	1.2
	0.8	-25.5	-15.6	4.4	-2.3	1.2	1.1
	1.0	-31.8	-43.1	-27.9	-0.0	-0.0	-0.0
1.0	0.	-0.0	-0.	0.0	-156.4	-62.3	-8.6
	0.2	-0.0	0.	0.0	-136.5	-69.4	-7.2
	0.4	0.0	0.	-0.0	-86.9	-75.0	-7.9
	0.6	0.0	-0.	-0.0	-33.2	-53.6	-14.5
	0.8	0.0	-0.	0.0	-2.7	-14.1	-13.9
	1.0	-0.0	0.	-0.0	-0.0	-0.0	0.0

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TABLE XXII. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 2.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	0.	0.	0.
	0.2	32.19	22.98	2.14	0.04	0.10	0.01
	0.4	24.96	42.89	4.46	0.05	0.13	0.02
	0.6	-8.45	38.89	7.18	0.03	0.09	0.01
	0.8	-11.60	-14.25	4.49	0.01	0.02	0.00
	1.0	-18.78	-70.46	-34.03	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	32.12	22.80	2.12	0.11	0.29	0.04
	0.4	24.88	42.68	4.43	0.15	0.41	0.06
	0.6	-8.47	38.82	7.17	0.11	0.30	0.04
	0.8	-11.56	-14.13	4.51	0.03	0.10	0.01
	1.0	-18.73	-70.34	-34.02	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	31.68	21.75	1.97	0.49	1.21	0.18
	0.4	24.37	41.33	4.23	0.66	1.77	0.27
	0.6	-8.63	38.21	7.08	0.47	1.40	0.21
	0.8	-11.31	-13.48	4.61	0.16	0.50	0.08
	1.0	-18.29	-69.16	-33.86	0.00	0.00	0.00
0.6	0.	0.	0.	0.	0.	0.	0.
	0.2	29.61	17.73	1.39	1.72	3.49	0.53
	0.4	22.10	35.68	3.38	2.12	5.37	0.85
	0.6	-9.25	34.86	6.54	1.28	4.63	0.78
	0.8	-10.34	-11.14	4.97	0.38	1.86	0.33
	1.0	-15.85	-62.34	-32.86	0.00	0.00	0.00
0.8	0.	-0.	0.	0.	-0.	0.	0.
	0.2	20.74	7.75	0.22	3.85	3.26	0.33
	0.4	14.11	19.48	1.05	3.34	6.13	0.94
	0.6	-8.92	22.04	4.10	0.21	6.51	1.59
	0.8	-6.88	-6.14	5.30	-0.60	3.01	1.23
	1.0	-8.16	-34.55	-27.03	0.00	0.00	0.00
1.0	0.	0.	0.	0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-32.78	-26.40	-2.62
	0.4	-0.00	-0.00	-0.00	-26.95	-50.55	-5.76
	0.6	-0.00	0.00	0.00	-0.50	-49.60	-12.03
	0.8	0.00	0.00	0.00	3.82	-16.13	-13.18
	1.0	-0.00	-0.00	-0.00	-0.00	-0.00	0.00

TABLE XXIII. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 2.50

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	121.1	41.1	4.4	2.5	1.4	0.2
	0.2	98.7	44.1	4.0	2.2	1.3	0.2
	0.4	40.5	42.7	4.0	1.6	0.9	0.1
	0.6	-31.0	13.3	5.0	0.8	0.5	0.1
	0.8	-95.3	-57.6	-0.7	0.2	0.1	0.0
	1.0	-160.0	-134.1	-41.9	0.0	0.0	0.0
0.2	0.	119.4	40.1	4.2	4.7	2.8	0.3
	0.2	97.2	43.3	3.9	4.3	2.5	0.3
	0.4	39.9	42.4	4.0	3.2	1.9	0.2
	0.6	-30.5	13.6	5.1	1.7	1.0	0.1
	0.8	-93.9	-56.8	-0.6	0.5	0.3	0.0
	1.0	-158.3	-133.1	-41.8	0.0	0.0	0.0
0.4	0.	111.8	35.6	3.7	12.0	7.2	0.9
	0.2	90.9	39.6	3.5	11.0	6.6	0.8
	0.4	37.1	40.7	3.8	8.3	5.0	0.6
	0.6	-28.5	14.7	5.2	4.7	2.9	0.4
	0.8	-87.3	-52.9	-0.1	1.4	0.9	0.1
	1.0	-149.4	-127.9	-41.1	0.0	0.0	0.0
0.6	0.	91.0	23.8	2.2	21.5	12.7	1.6
	0.2	73.3	29.3	2.2	19.7	11.9	1.6
	0.4	28.3	35.1	3.1	14.9	9.6	1.3
	0.6	-24.4	16.6	5.4	8.6	5.9	0.8
	0.8	-69.0	-41.8	1.3	2.7	2.0	0.3
	1.0	-118.9	-109.3	-38.8	0.0	0.0	0.0
0.8	0.	48.1	4.5	0.3	8.5	1.8	0.0
	0.2	37.4	10.8	0.1	6.9	2.8	0.3
	0.4	11.1	20.8	0.9	3.5	4.7	0.9
	0.6	-16.6	14.6	4.6	0.7	4.9	1.3
	0.8	-33.8	-20.4	4.0	-0.1	2.3	0.9
	1.0	-48.7	-58.6	-31.2	0.0	0.0	0.0
1.0	0.	0.0	0.0	0.0	-156.1	-62.1	-8.5
	0.2	0.0	0.0	0.0	-136.3	-69.3	-7.2
	0.4	-0.0	-0.0	-0.0	-86.7	-74.9	-7.9
	0.6	-0.0	-0.0	-0.0	-33.1	-53.5	-14.5
	0.8	0.0	0.0	0.0	-2.7	-14.0	-13.9
	1.0	-0.0	-0.0	-0.0	-0.0	-0.0	0.0

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TABLE XXIV. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 2.50

x/β	y	Θ_1x10^3	Θ_2x10^3	Θ_3x10^3	Φ_1x10^3	Φ_2x10^3	Φ_3x10^3
0.	0.	0.	0.	0.	-0.	-0.	-0.
	0.2	32.21	23.02	2.15	0.00	0.00	0.00
	0.4	24.97	42.93	4.46	0.00	0.00	-0.00
	0.6	-8.45	38.88	7.17	-0.00	-0.00	-0.00
	0.8	-11.61	-14.28	4.49	-0.00	-0.00	-0.00
	1.0	-18.76	-70.41	-34.02	0.00	0.00	0.00
0.2	0.	0.	0.	0.	0.	0.	0.
	0.2	32.20	23.00	2.15	0.02	0.05	0.01
	0.4	24.96	42.90	4.46	0.02	0.06	0.01
	0.6	-8.45	38.88	7.17	0.02	0.04	0.01
	0.8	-11.60	-14.26	4.49	0.00	0.01	0.00
	1.0	-18.76	-70.43	-34.02	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	32.07	22.67	2.10	0.16	0.43	0.06
	0.4	24.82	42.51	4.40	0.22	0.60	0.09
	0.6	-8.49	38.75	7.16	0.16	0.46	0.07
	0.8	-11.52	-14.05	4.52	0.05	0.15	0.02
	1.0	-18.68	-70.21	-34.00	0.00	0.00	0.00
0.6	0.	0.	0.	0.	0.	0.	0.
	0.2	31.02	20.32	1.76	0.95	2.19	0.33
	0.4	23.62	39.39	3.94	1.24	3.26	0.51
	0.6	-8.87	37.15	6.91	0.84	2.68	0.43
	0.8	-10.98	-12.62	4.74	0.28	1.02	0.16
	1.0	-17.52	-67.07	-33.56	0.00	0.00	0.00
0.8	0.	0.	0.	0.	-0.	0.	0.
	0.2	24.20	10.79	0.51	3.49	4.39	0.58
	0.4	17.00	24.78	1.77	3.48	7.45	1.20
	0.6	-9.46	26.68	5.03	1.06	7.27	1.51
	0.8	-8.19	-7.55	5.37	-0.07	3.26	0.91
	1.0	-10.60	-44.70	-29.62	0.00	0.00	0.00
1.0	0.	0.	0.	0.	0.	-0.	-0.
	0.2	0.00	0.00	0.00	-32.78	-26.40	-2.62
	0.4	0.00	-0.00	0.00	-26.95	-50.55	-5.76
	0.6	-0.00	-0.00	-0.00	-0.50	-47.60	-12.03
	0.8	0.00	0.00	0.00	3.82	-16.13	-13.18
	1.0	-0.00	-0.00	-0.00	-0.00	-0.00	0.00

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TABLE XXV. - Θ AND Φ FUNCTIONS FOR EVEN TEMPERATURE DISTRIBUTIONS; β = 3.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	121.7	41.4	4.4	0.3	0.2	0.0
	0.2	99.0	44.3	4.0	0.3	0.2	0.0
	0.4	40.4	42.7	4.0	0.2	0.1	0.0
	0.6	-31.4	13.0	5.0	0.1	0.0	0.0
	0.8	-95.6	-57.8	-0.7	0.0	0.0	0.0
	1.0	-159.5	-133.7	-41.8	0.0	0.0	0.0
0.2	0.	121.1	41.1	4.4	1.6	0.9	0.1
	0.2	98.6	44.1	4.0	1.4	0.8	0.1
	0.4	40.3	42.6	4.0	1.0	0.6	0.1
	0.6	-31.1	13.2	5.0	0.5	0.3	0.0
	0.8	-95.2	-57.6	-0.6	0.1	0.1	0.0
	1.0	-159.3	-133.7	-41.8	0.0	0.0	0.0
0.4	0.	117.2	38.8	4.1	7.0	4.1	0.5
	0.2	95.4	42.2	3.8	6.4	3.8	0.5
	0.4	39.1	41.9	3.9	4.7	2.8	0.4
	0.6	-29.9	13.9	5.1	2.6	1.6	0.2
	0.8	-92.0	-55.7	-0.4	0.8	0.5	0.1
	1.0	-155.8	-131.7	-41.6	0.0	0.0	0.0
0.6	0.	101.4	29.6	2.9	18.2	10.9	1.4
	0.2	82.2	34.4	2.8	16.7	10.1	1.3
	0.4	32.8	38.1	3.5	12.7	7.9	1.0
	0.6	-26.3	15.9	5.4	7.2	4.7	0.6
	0.8	-78.2	-47.4	0.6	2.3	1.5	0.2
	1.0	-134.9	-119.2	-40.0	0.0	0.0	0.0
0.8	0.	58.7	8.3	0.6	15.9	7.3	0.8
	0.2	46.1	14.7	0.5	14.1	7.7	1.0
	0.4	14.9	24.6	1.5	9.5	7.9	1.3
	0.6	-18.7	15.9	5.0	4.8	6.4	1.3
	0.8	-41.9	-25.3	3.5	1.4	2.7	0.7
	1.0	-65.6	-72.4	-33.6	0.0	0.0	0.0
1.0	0.	0.0	-0.0	0.0	-156.1	-62.1	-8.5
	0.2	0.0	0.0	0.0	-136.3	-69.2	-7.2
	0.4	-0.0	0.0	-0.0	-86.7	-74.9	-7.9
	0.6	-0.0	0.0	-0.0	-33.1	-53.5	-14.5
	0.8	0.0	-0.0	0.0	-2.7	-14.0	-13.9
	1.0	-0.0	0.0	-0.0	-0.0	-0.0	0.0

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TABLE XXVI. - Θ AND Φ FUNCTIONS FOR ODD TEMPERATURE DISTRIBUTIONS; β = 3.00

x/β	y	$\Theta_1 \times 10^3$	$\Theta_2 \times 10^3$	$\Theta_3 \times 10^3$	$\Phi_1 \times 10^3$	$\Phi_2 \times 10^3$	$\Phi_3 \times 10^3$
0.	0.	0.	0.	0.	-0.	-0.	-0.
	0.2	32.21	23.02	2.15	-0.00	-0.00	-0.00
	0.4	24.97	42.92	4.46	-0.00	-0.00	-0.00
	0.6	-8.46	38.87	7.17	-0.00	-0.00	-0.00
	0.8	-11.61	-14.28	4.49	-0.00	-0.00	-0.00
	1.0	-18.75	-70.39	-34.02	-0.00	-0.00	-0.00
0.2	0.	0.	0.	0.	-0.	-0.	-0.
	0.2	32.21	23.02	2.15	0.00	0.00	0.00
	0.4	24.97	42.92	4.46	0.00	0.00	0.00
	0.6	-8.45	38.87	7.17	0.00	0.00	0.00
	0.8	-11.61	-14.28	4.49	-0.00	-0.00	-0.00
	1.0	-18.76	-70.41	-34.02	0.00	0.00	0.00
0.4	0.	0.	0.	0.	0.	0.	0.
	0.2	32.18	22.94	2.14	0.05	0.12	0.02
	0.4	24.94	42.84	4.45	0.06	0.17	0.02
	0.6	-8.45	38.87	7.17	0.04	0.12	0.02
	0.8	-11.59	-14.22	4.50	0.01	0.04	0.01
	1.0	-18.76	-70.41	-34.02	0.00	0.00	0.00
0.6	0.	0.	0.	0.	0.	0.	0.
	0.2	31.68	21.75	1.97	0.49	1.21	0.18
	0.4	24.37	41.32	4.23	0.66	1.77	0.27
	0.6	-8.63	38.21	7.08	0.47	1.40	0.21
	0.8	-11.31	-13.48	4.61	0.16	0.51	0.08
	1.0	-18.29	-69.16	-33.86	0.00	0.00	0.00
0.8	0.	0.	0.	0.	0.	0.	0.
	0.2	26.67	13.52	0.83	2.88	4.51	0.65
	0.4	19.22	29.25	2.41	3.15	7.32	1.19
	0.6	-9.56	30.27	5.72	1.40	6.77	1.28
	0.8	-9.15	-8.90	5.27	0.24	2.93	0.65
	1.0	-12.75	-52.53	-31.21	0.00	0.00	0.00
1.0	0.	0.	-0.	-0.	0.	-0.	-0.
	0.2	0.00	-0.00	-0.00	-32.78	-26.40	-2.62
	0.4	-0.00	-0.00	-0.00	-26.95	-50.55	-5.76
	0.6	-0.00	0.00	0.00	-0.50	-49.60	-12.03
	0.8	0.00	-0.00	0.00	3.82	-16.13	-13.18
	1.0	-0.00	0.00	-0.00	-0.00	-0.00	0.00

TABLE XXVII. - CONSTANT COEFFICIENTS OF STRESS FUNCTIONS IN x FOR EVEN TEMPERATURE DISTRIBUTIONS

i	y_i	g_i	h_i	j	a_{ij}	b_{ij}	c_{ij}	d_{ij}	e_{ij}	f_{ij}	l_{ij}
1	1/6	2.106	1.125	1	1	0	2.106	-1.125	3.168	-4.738	-0.02382
				2	1	0	5.423	-1.278	27.77	-13.86	-.01383
				3	1	0	9.235	-3.371	73.92	-62.27	-.001735
2	3/6	5.423	1.278	1	0.5637	0.03947	1.231	-0.5511	1.973	-2.546	-0.01310
				2	-1.236	.2541	-6.379	2.958	-30.81	24.20	-.009052
				3	-2.968	1.061	-23.83	19.80	-153.3	263.2	-.001279
3	5/6	9.235	3.371	1	0.08273	0.01802	0.1945	-0.05513	0.3475	-0.3349	-0.001931
				2	-.4420	-.04044	-2.448	.3457	-12.84	5.005	-.001609
				3	2.019	-4.496	3.487	-48.33	-130.8	-458.1	-.0003497

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TABLE XXVIII. - CONSTANT COEFFICIENTS OF STRESS FUNCTIONS

IN x FOR ODD TEMPERATURE DISTRIBUTIONS

i	y_i	g_i	h_i	j	a_{ij}	b_{ij}	c_{ij}	d_{ij}	e_{ij}	f_{ij}	l_{ij}
1	1/6	3.730	1.385	1	1	0	3.730	-1.385	12.00	-10.34	-0.001133
				2	1	0	9.124	-2.222	78.30	-40.55	-.001825
				3	1	1	6.282	8.934	39.46	79.82	-.0002539
2	3/6	9.124	2.222	1	1.540	0.1781	5.992	-1.469	20.32	-13.78	-0.001083
				2	-.9826	.6484	-7.524	8.099	-50.65	90.61	-.003393
				3	-.5271	-.8553	-3.311	-7.642	-20.80	-68.27	-.0005603
3	5/6	6.282	8.934	1	0.3003	0.1116	1.275	0.000438	4.756	-1.764	-0.00008860
				2	-.006560	-1.509	-3.413	-13.75	-61.70	-117.0	-.0008588
				3	-.3783	.2921	-2.376	2.610	-14.93	23.32	-.0002450

TABLE XXIX. - COEFFICIENTS OF STRESS FUNCTIONS IN y FOR EVEN TEMPERATURE DISTRIBUTIONS

j	k	r_{jk}	s_{jk}	t_{jk}
1	1	1.240	-18.45	-18.45
	2	-9.224	87.48	262.4
	3	21.87	-126.2	-630.8
	4	-21.63	57.13	399.9
	5	7.141	------	-------
2	1	-0.3472	27.39	27.39
	2	13.69	-177.4	-532.2
	3	-44.35	294.0	1470
	4	49.00	-144.0	-1008
	5	-18.00	-------	--------
3	1	0.2510	-21.09	-21.09
	2	-10.54	225.9	677.8
	3	56.48	-434.0	-2470
	4	-82.34	289.2	2024
	5	36.15	-------	--------

TABLE XXX. - COEFFICIENTS OF STRESS FUNCTIONS IN y FOR ODD TEMPERATURE DISTRIBUTIONS

j	k	r_{jk}	s_{jk}	t_{jk}
1	1	7.439	7.439	-332.1
	2	-55.34	-166.0	2624
	3	131.2	656.1	5299
	4	-126.2	-883.1	3085
	5	42.85	385.6	------
2	1	-0.6944	-0.6944	164.3
	2	27.39	82.17	-1774
	3	-88.69	-443.5	4116
	4	98.00	686.0	-2592
	5	-36.00	-324.0	-------
3	1	0.3012	0.312	-75.91
	2	-12.65	-37.96	1356
	3	67.78	338.9	-4150
	4	-98.81	-691.6	3123
	5	43.38	390.4	--------

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TABLE XXXI. - VARIABLE COEFFICIENTS OF STRESS FUNCTIONS IN x FOR EVEN TEMPERATURE DISTRIBUTIONS

β	j	A_{jk}			B_{jk}		
		$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
0.10	1	2.316×10^{-2}	6.458×10^{-4}	3.056×10^{-6}	-1.678×10^{-2}	-7.760×10^{-4}	5.130×10^{-6}
	2	1.447×10^{-2}	-6.338×10^{-4}	-1.190×10^{-5}	-9.056×10^{-3}	1.180×10^{-3}	-1.239×10^{-5}
	3	1.929×10^{-3}	-2.179×10^{-4}	2.414×10^{-5}	-8.569×10^{-4}	1.384×10^{-4}	4.539×10^{-6}
0.20	1	2.316×10^{-2}	5.970×10^{-4}	9.297×10^{-7}	-1.595×10^{-2}	-6.258×10^{-4}	3.792×10^{-6}
	2	1.443×10^{-2}	-6.024×10^{-4}	-5.248×10^{-6}	-8.580×10^{-3}	9.653×10^{-4}	-9.953×10^{-6}
	3	1.914×10^{-3}	-1.910×10^{-4}	1.300×10^{-5}	-8.341×10^{-4}	1.286×10^{-4}	5.770×10^{-6}
0.30	1	2.304×10^{-2}	5.016×10^{-4}	-2.926×10^{-7}	-1.456×10^{-2}	-4.280×10^{-4}	1.974×10^{-6}
	2	1.428×10^{-2}	-5.116×10^{-4}	-9.375×10^{-7}	-7.820×10^{-3}	6.762×10^{-4}	-5.993×10^{-6}
	3	1.880×10^{-3}	-1.497×10^{-4}	4.565×10^{-6}	-7.939×10^{-4}	1.020×10^{-4}	3.852×10^{-6}
0.40	1	2.265×10^{-2}	3.819×10^{-4}	-4.786×10^{-7}	-1.266×10^{-2}	-2.503×10^{-4}	7.041×10^{-7}
	2	1.395×10^{-2}	-3.807×10^{-4}	4.590×10^{-7}	-6.814×10^{-3}	4.103×10^{-4}	-2.902×10^{-6}
	3	1.822×10^{-3}	-1.052×10^{-4}	1.230×10^{-6}	-7.106×10^{-4}	6.301×10^{-5}	1.940×10^{-6}
0.50	1	2.185×10^{-2}	2.696×10^{-4}	-2.995×10^{-7}	-1.038×10^{-2}	-1.279×10^{-4}	1.342×10^{-7}
	2	1.337×10^{-2}	-2.552×10^{-4}	5.860×10^{-7}	-5.575×10^{-3}	2.232×10^{-4}	-1.183×10^{-6}
	3	1.731×10^{-3}	-6.786×10^{-5}	2.140×10^{-7}	-5.858×10^{-4}	3.225×10^{-5}	8.570×10^{-7}
0.75	1	1.775×10^{-2}	9.327×10^{-5}	-1.576×10^{-8}	-4.238×10^{-3}	-9.308×10^{-6}	-3.410×10^{-8}
	2	1.072×10^{-2}	-7.478×10^{-5}	1.428×10^{-7}	-2.141×10^{-3}	3.614×10^{-5}	-5.083×10^{-8}
	3	1.362×10^{-3}	-1.874×10^{-5}	-4.672×10^{-8}	-1.999×10^{-4}	2.816×10^{-6}	7.008×10^{-8}
1.00	1	1.202×10^{-2}	2.639×10^{-5}	3.078×10^{-9}	1.935×10^{-4}	6.052×10^{-6}	-3.789×10^{-9}
	2	7.181×10^{-3}	-1.961×10^{-5}	1.423×10^{-8}	3.545×10^{-4}	3.263×10^{-6}	7.033×10^{-9}
	3	8.991×10^{-4}	-4.610×10^{-6}	-8.157×10^{-9}	8.615×10^{-5}	-7.691×10^{-7}	1.141×10^{-9}
1.25	1	7.042×10^{-3}	6.258×10^{-6}	5.378×10^{-10}	2.080×10^{-3}	3.782×10^{-6}	3.990×10^{-12}
	2	4.162×10^{-3}	-4.890×10^{-6}	4.508×10^{-10}	1.377×10^{-3}	-6.926×10^{-7}	1.536×10^{-9}
	3	5.136×10^{-4}	-1.044×10^{-6}	-6.201×10^{-10}	1.960×10^{-4}	-5.497×10^{-7}	-5.279×10^{-10}
1.50	1	3.680×10^{-3}	1.244×10^{-6}	3.601×10^{-11}	2.339×10^{-3}	1.450×10^{-6}	4.127×10^{-11}
	2	2.150×10^{-3}	-1.130×10^{-6}	-8.371×10^{-11}	1.466×10^{-3}	-5.556×10^{-7}	1.356×10^{-10}
	3	2.608×10^{-4}	-2.097×10^{-7}	-1.777×10^{-12}	1.956×10^{-4}	-2.179×10^{-7}	-8.084×10^{-11}
1.75	1	1.703×10^{-3}	1.879×10^{-7}	-6.598×10^{-13}	1.928×10^{-3}	4.563×10^{-7}	5.413×10^{-12}
	2	9.784×10^{-4}	-2.310×10^{-7}	-1.558×10^{-11}	1.183×10^{-3}	-2.270×10^{-7}	2.763×10^{-12}
	3	1.158×10^{-4}	-3.363×10^{-8}	5.882×10^{-12}	1.536×10^{-4}	-7.026×10^{-8}	-5.476×10^{-12}
2.00	1	6.492×10^{-4}	9.053×10^{-9}	-4.435×10^{-13}	1.372×10^{-3}	1.268×10^{-7}	3.097×10^{-13}
	2	3.606×10^{-4}	-3.814×10^{-8}	-1.234×10^{-12}	8.307×10^{-4}	-7.430×10^{-8}	-9.730×10^{-13}
	3	4.042×10^{-5}	-2.537×10^{-9}	7.953×10^{-13}	1.060×10^{-4}	-1.992×10^{-8}	7.434×10^{-14}
2.50	1	-6.375×10^{-5}	-4.541×10^{-9}	-2.538×10^{-15}	5.254×10^{-4}	7.118×10^{-9}	-4.689×10^{-15}
	2	-4.818×10^{-5}	9.132×10^{-10}	1.094×10^{-14}	3.120×10^{-4}	-5.477×10^{-9}	-1.100×10^{-14}
	3	-7.800×10^{-6}	6.547×10^{-10}	-1.630×10^{-15}	3.878×10^{-5}	-1.164×10^{-9}	7.719×10^{-15}
3.00	1	-1.166×10^{-4}	-5.243×10^{-10}	4.886×10^{-17}	1.432×10^{-4}	1.996×10^{-10}	-1.959×10^{-17}
	2	-7.229×10^{-5}	2.658×10^{-10}	9.551×10^{-17}	8.314×10^{-5}	-2.559×10^{-10}	1.198×10^{-16}
	3	-9.519×10^{-6}	8.104×10^{-11}	-7.388×10^{-17}	9.994×10^{-6}	-3.611×10^{-11}	-2.473×10^{-17}

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TABLE XXXII. - VARIABLE COEFFICIENTS OF STRESS FUNCTIONS IN x FOR ODD TEMPERATURE DISTRIBUTIONS

β	j	A_{jk}			B_{jk}		
		k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
0.10	1	7.010×10^{-4}	-3.653×10^{-5}	9.622×10^{-4}	-9.514×10^{-4}	1.864×10^{-4}	-4.978×10^{-4}
	2	1.877×10^{-3}	-2.314×10^{-5}	-3.913×10^{-4}	-2.641×10^{-3}	-1.683×10^{-4}	3.624×10^{-4}
	3	3.386×10^{-4}	1.127×10^{-4}	-6.988×10^{-5}	-3.033×10^{-4}	7.787×10^{-5}	-1.274×10^{-4}
0.20	1	6.974×10^{-4}	-4.655×10^{-5}	7.613×10^{-4}	-8.602×10^{-4}	1.149×10^{-4}	-3.265×10^{-4}
	2	1.878×10^{-3}	4.954×10^{-6}	-3.012×10^{-4}	-2.374×10^{-3}	-1.101×10^{-4}	2.358×10^{-4}
	3	3.262×10^{-4}	5.814×10^{-5}	-6.546×10^{-5}	-2.868×10^{-4}	5.084×10^{-5}	-6.385×10^{-5}
0.30	1	6.704×10^{-4}	-3.690×10^{-5}	5.082×10^{-4}	-7.104×10^{-4}	5.191×10^{-5}	-1.702×10^{-4}
	2	1.816×10^{-3}	1.406×10^{-5}	-1.888×10^{-4}	-1.958×10^{-3}	-5.646×10^{-5}	1.241×10^{-4}
	3	3.028×10^{-4}	2.124×10^{-5}	-4.850×10^{-5}	-2.502×10^{-4}	2.414×10^{-5}	-2.157×10^{-5}
0.40	1	6.075×10^{-4}	-2.134×10^{-5}	3.013×10^{-4}	-5.280×10^{-4}	1.855×10^{-5}	-7.716×10^{-5}
	2	1.653×10^{-3}	1.124×10^{-5}	-1.015×10^{-4}	-1.459×10^{-3}	-2.492×10^{-5}	5.761×10^{-5}
	3	2.672×10^{-4}	6.558×10^{-6}	-2.838×10^{-5}	-1.903×10^{-4}	1.055×10^{-5}	-7.070×10^{-6}
0.50	1	5.135×10^{-4}	-1.043×10^{-5}	1.683×10^{-4}	-3.517×10^{-4}	5.387×10^{-6}	-3.272×10^{-5}
	2	1.402×10^{-3}	6.786×10^{-6}	-5.062×10^{-5}	-9.751×10^{-4}	-9.869×10^{-6}	2.495×10^{-5}
	3	2.215×10^{-4}	1.694×10^{-6}	-1.499×10^{-5}	-1.261×10^{-4}	4.507×10^{-6}	-2.562×10^{-6}
0.75	1	2.632×10^{-4}	-1.207×10^{-6}	3.618×10^{-5}	-7.454×10^{-5}	-1.266×10^{-7}	-3.561×10^{-6}
	2	7.213×10^{-4}	1.173×10^{-6}	-8.962×10^{-6}	-2.097×10^{-4}	-5.977×10^{-7}	2.754×10^{-6}
	3	1.097×10^{-4}	-8.645×10^{-8}	-2.904×10^{-6}	-2.254×10^{-5}	4.654×10^{-7}	-2.577×10^{-7}
1.00	1	1.091×10^{-4}	-9.772×10^{-8}	7.559×10^{-6}	7.314×10^{-6}	-7.669×10^{-8}	-3.821×10^{-7}
	2	2.998×10^{-4}	1.353×10^{-7}	-1.781×10^{-6}	1.806×10^{-5}	9.724×10^{-9}	2.948×10^{-7}
	3	4.429×10^{-5}	-3.241×10^{-8}	-5.918×10^{-7}	6.211×10^{-6}	3.551×10^{-8}	-2.766×10^{-8}
1.25	1	3.944×10^{-5}	-4.347×10^{-9}	1.573×10^{-6}	1.728×10^{-5}	-1.193×10^{-8}	-4.095×10^{-8}
	2	1.087×10^{-4}	1.123×10^{-8}	-3.681×10^{-7}	4.674×10^{-5}	8.119×10^{-9}	3.155×10^{-8}
	3	1.558×10^{-5}	-4.726×10^{-9}	-1.228×10^{-7}	8.216×10^{-6}	1.334×10^{-9}	-2.968×10^{-9}
1.50	1	1.229×10^{-5}	2.656×10^{-10}	3.271×10^{-7}	1.166×10^{-5}	-1.270×10^{-9}	-4.388×10^{-9}
	2	3.398×10^{-5}	5.378×10^{-10}	-7.653×10^{-8}	3.181×10^{-5}	1.310×10^{-9}	3.380×10^{-9}
	3	4.668×10^{-6}	-4.822×10^{-10}	-2.553×10^{-8}	5.122×10^{-6}	-1.389×10^{-10}	-3.181×10^{-10}
1.75	1	2.991×10^{-6}	9.152×10^{-11}	6.802×10^{-8}	5.959×10^{-6}	-9.598×10^{-11}	-4.701×10^{-10}
	2	8.326×10^{-6}	-2.392×10^{-11}	-1.592×10^{-8}	1.631×10^{-5}	1.427×10^{-10}	3.622×10^{-10}
	3	1.044×10^{-6}	-3.439×10^{-11}	-5.309×10^{-9}	2.519×10^{-6}	-3.805×10^{-11}	-3.408×10^{-11}
2.00	1	3.110×10^{-7}	1.312×10^{-11}	1.414×10^{-8}	2.606×10^{-6}	-3.401×10^{-12}	-5.037×10^{-11}
	2	9.027×10^{-7}	-9.769×10^{-12}	-3.310×10^{-9}	7.151×10^{-6}	1.110×10^{-11}	3.880×10^{-11}
	3	4.977×10^{-8}	-9.350×10^{-13}	-1.104×10^{-9}	1.072×10^{-6}	-5.157×10^{-12}	-3.652×10^{-12}
2.50	1	-2.206×10^{-7}	9.261×10^{-14}	6.117×10^{-10}	3.413×10^{-7}	1.070×10^{-13}	-5.782×10^{-13}
	2	-5.997×10^{-7}	-1.492×10^{-13}	-1.431×10^{-10}	9.416×10^{-7}	-3.997×10^{-14}	4.454×10^{-13}
	3	-1.001×10^{-7}	4.393×10^{-14}	-4.774×10^{-11}	1.327×10^{-7}	-3.264×10^{-14}	-4.192×10^{-14}
3.00	1	-6.005×10^{-8}	-5.723×10^{-16}	2.645×10^{-11}	1.886×10^{-8}	1.362×10^{-15}	-6.638×10^{-15}
	2	-1.646×10^{-7}	-3.171×10^{-16}	-6.190×10^{-12}	5.292×10^{-8}	-1.581×10^{-15}	5.113×10^{-15}
	3	-2.505×10^{-8}	5.090×10^{-16}	-2.065×10^{-12}	5.916×10^{-9}	2.598×10^{-16}	-4.813×10^{-16}

TABLE XXXIII. - COMPARISON OF CALCULATED WITH EXACT DIMENSIONLESS STRESS ALONG CHORD FAR FROM END OF SEMI-INFINITE PLATE FOR PARABOLIC CHORD-WISE TEMPERATURE DISTRIBUTION

y	Θ		Φ	
	Exact	Calaculated	Exact	Calculated
0	0.333	0.335	0	0.0010
.2	.293	.295	0	.0010
.4	.173	.174	0	.0006
.6	-.027	-.027	0	.0002
.8	-.307	-.308	0	.0000
1.0	-.667	-.670	0	.0000

E-1338

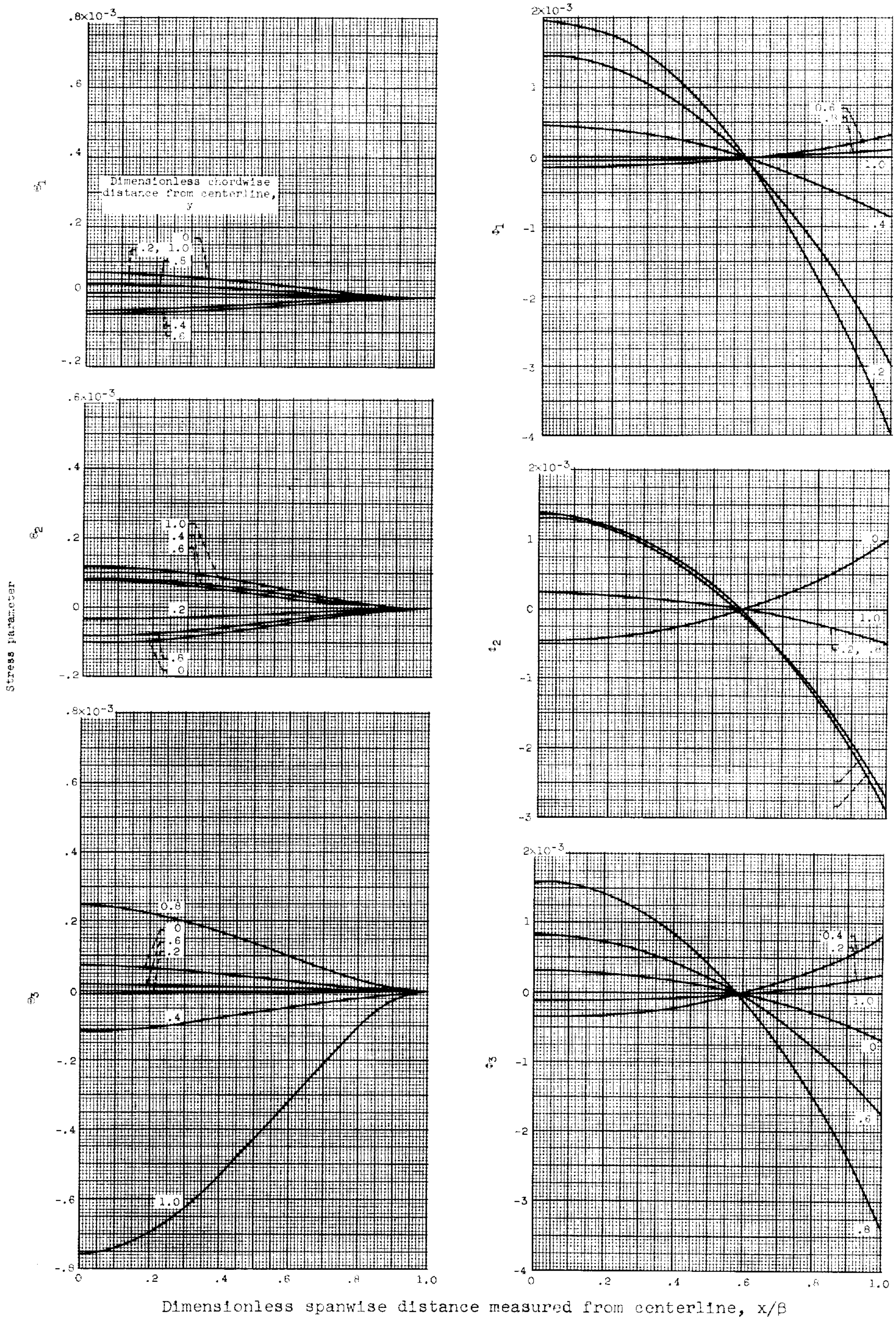

Figure 1. - Stress due to even thermal gradient for span-chord ratio β = 0.10.

E-1338

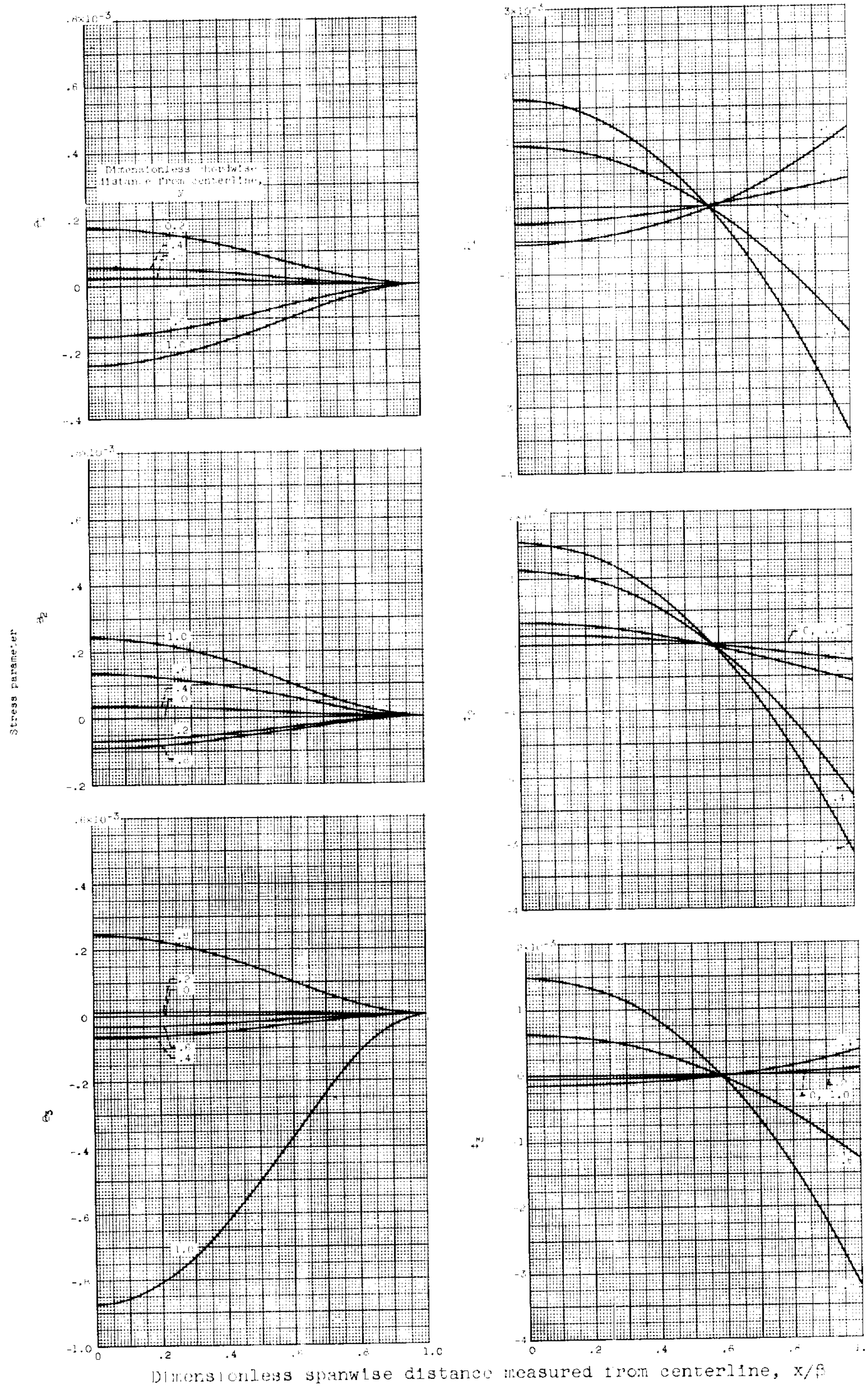

Figure 2. - Stress due to odd thermal gradient for span-chord ratio β = 0.10.

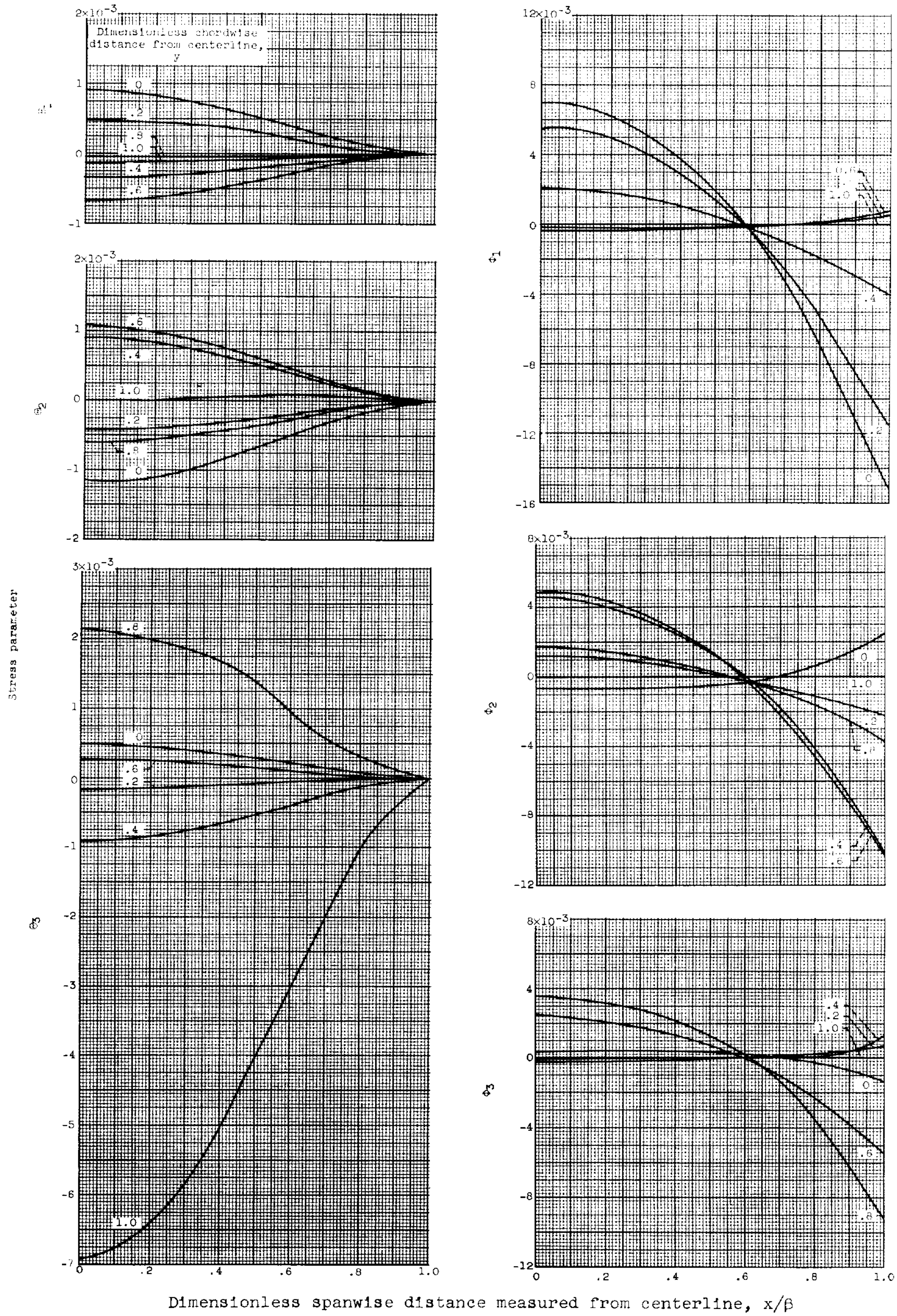

Figure 3. - Stress due to even thermal gradient for span-chord ratio β = 0.20.

E-1338

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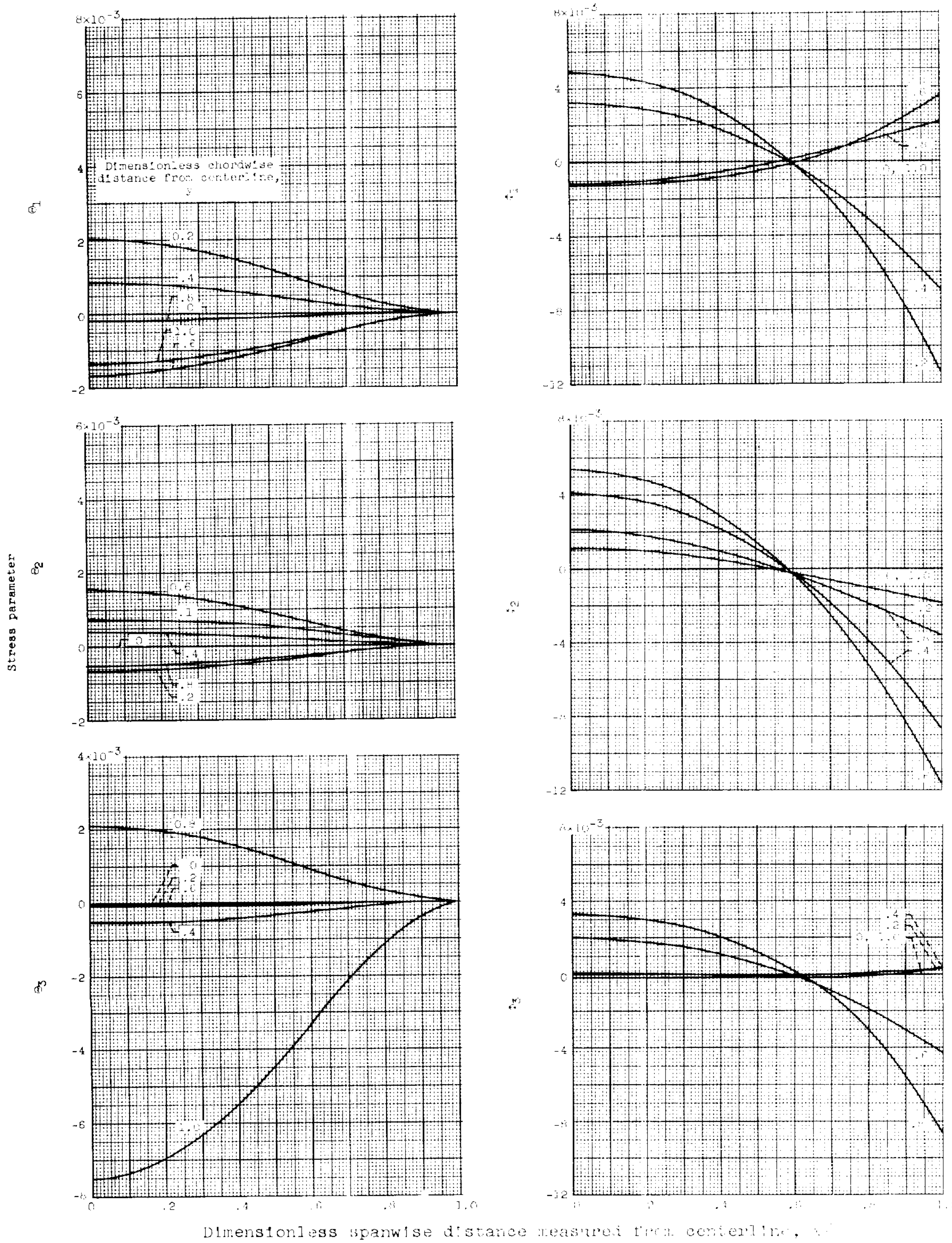
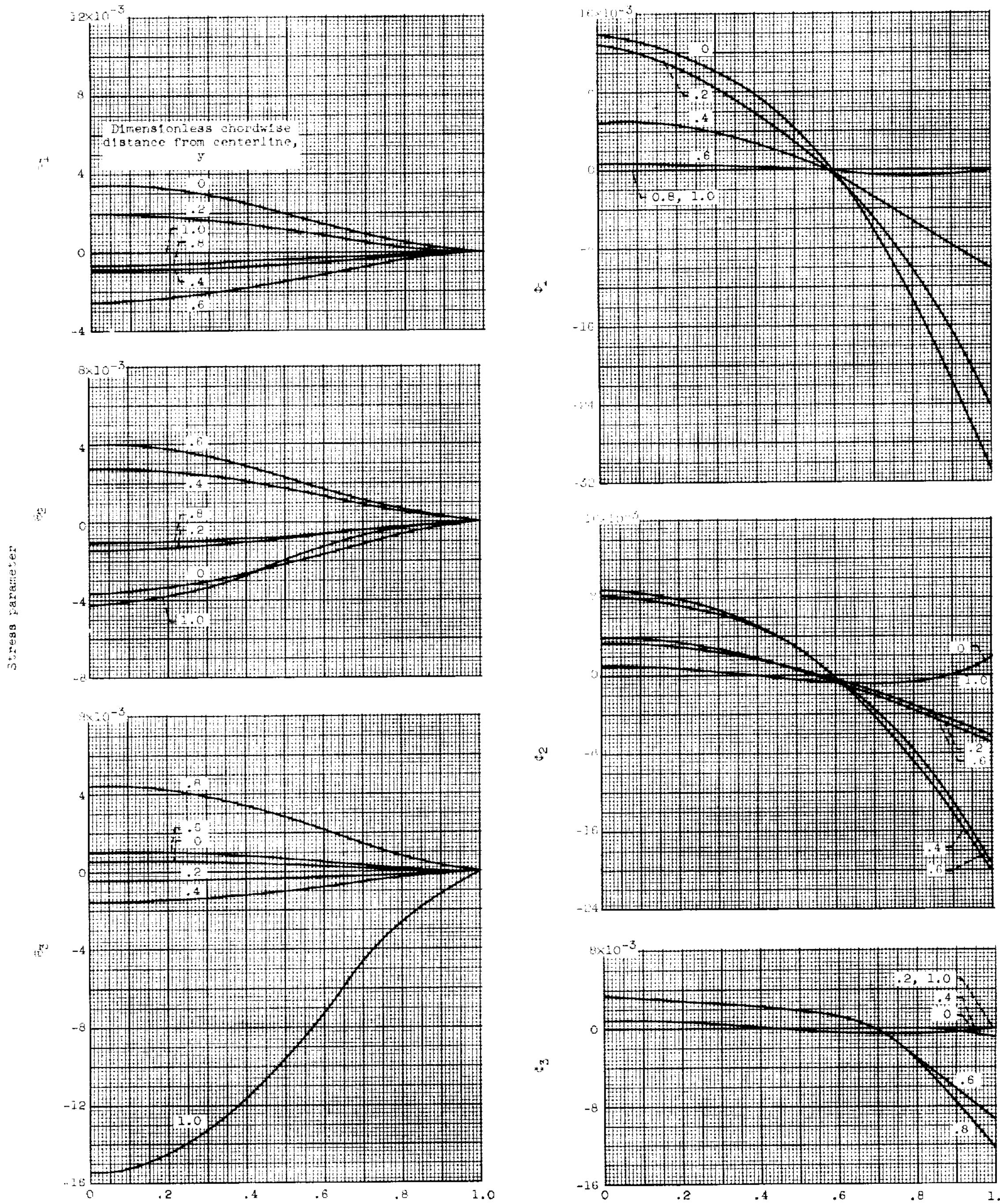


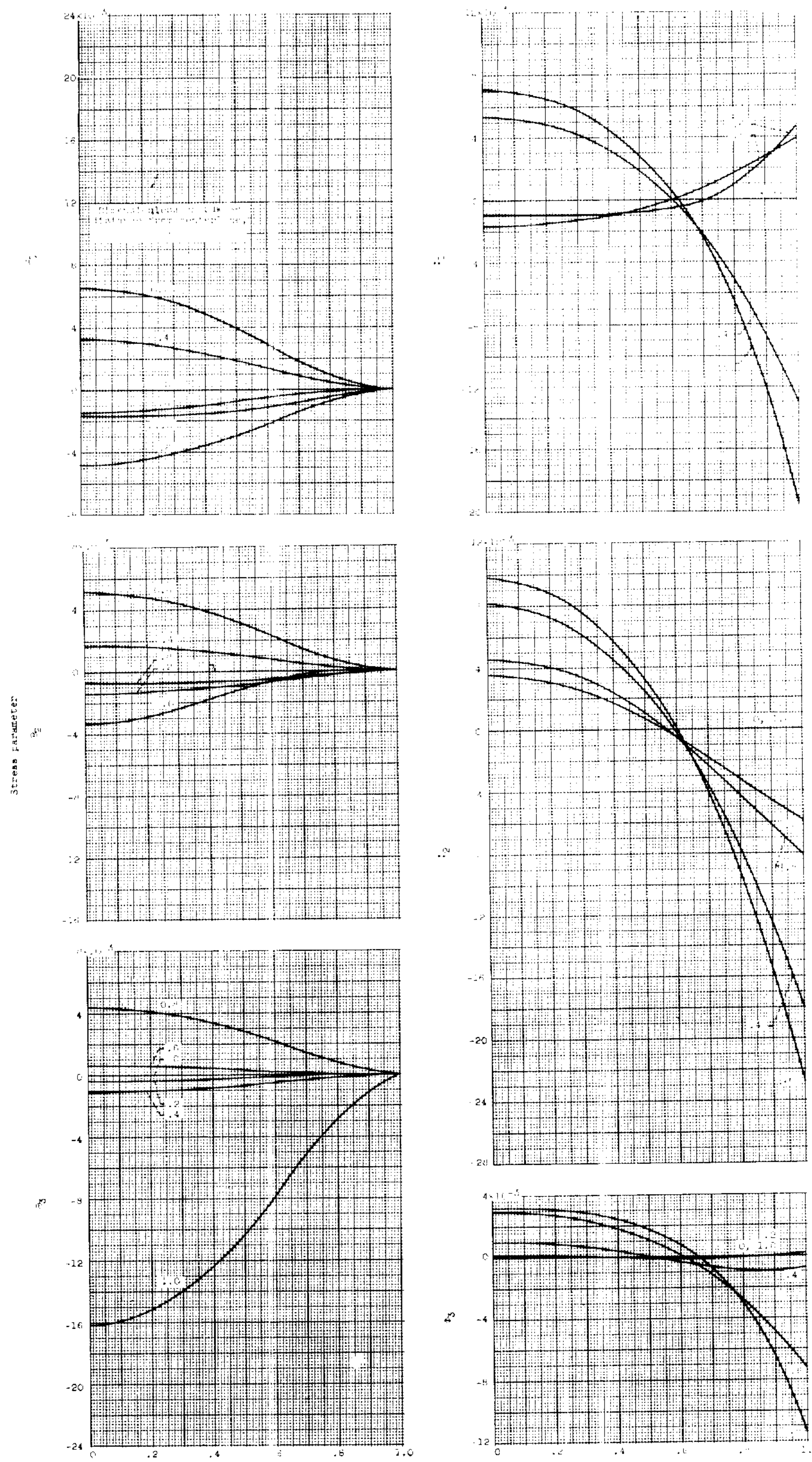
Figure 4. - Stress due to odd thermal gradient for span-chord ratio, [illegible] = [illegible].



Dimensionless spanwise distance measured from centerline, x/β

Figure 5. - Stress due to even thermal gradient for span-chord ratio β = 0.30.

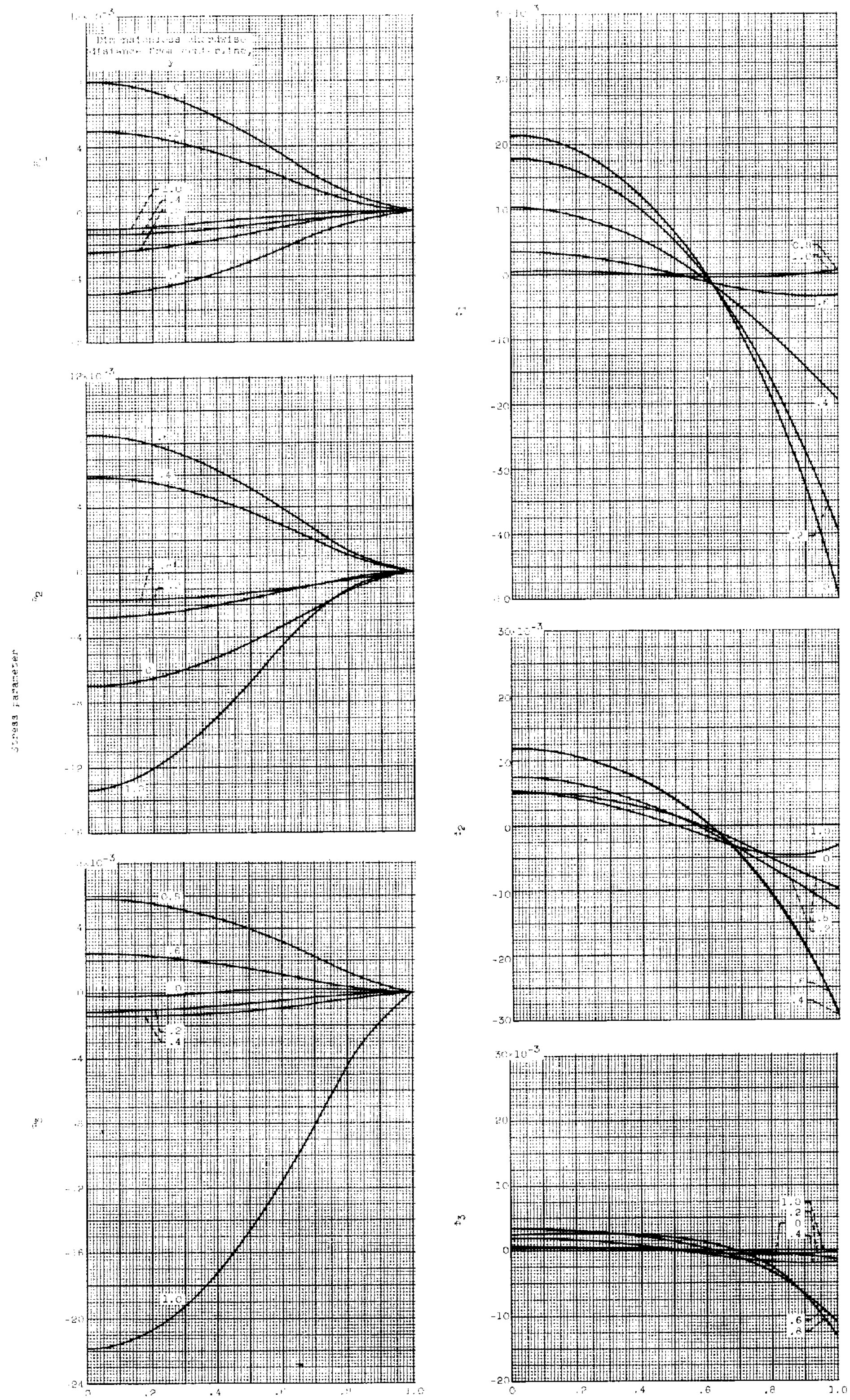
E-1338



Dimensionless spanwise distance measured from centerline, x/b

Figure 6. - Stress due to odd thermal gradient for span-chord ratio β = 0.30.

E-1338



Dimensionless spanwise distance measured from centerline, x/β

Figure 7. - Stress due to even thermal gradient for span-chord ratio β = 0.40.

E-1338

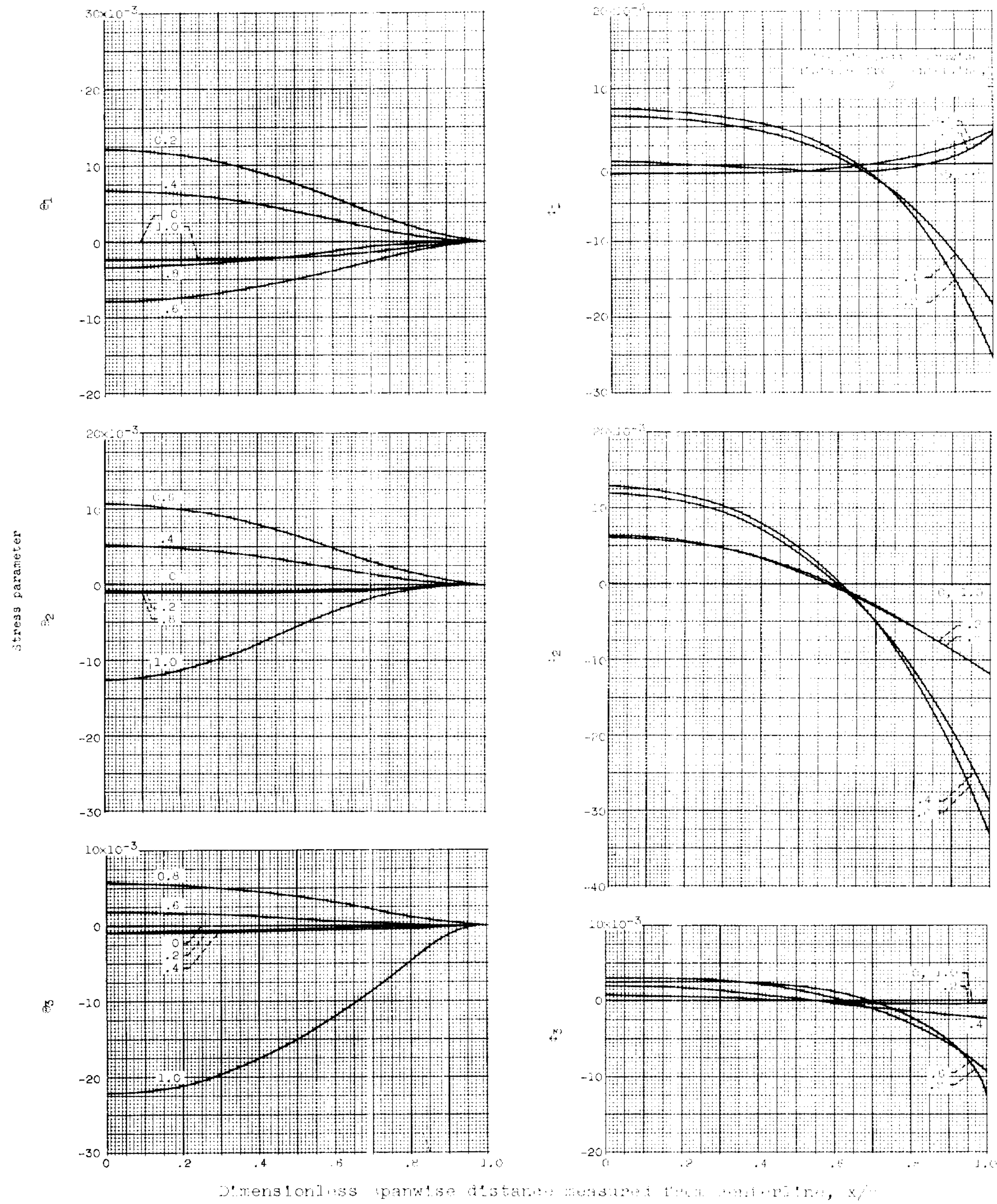


Figure [illegible]. - Stress due to odd thermal gradient for span-chord ratio [illegible] = 0.40.

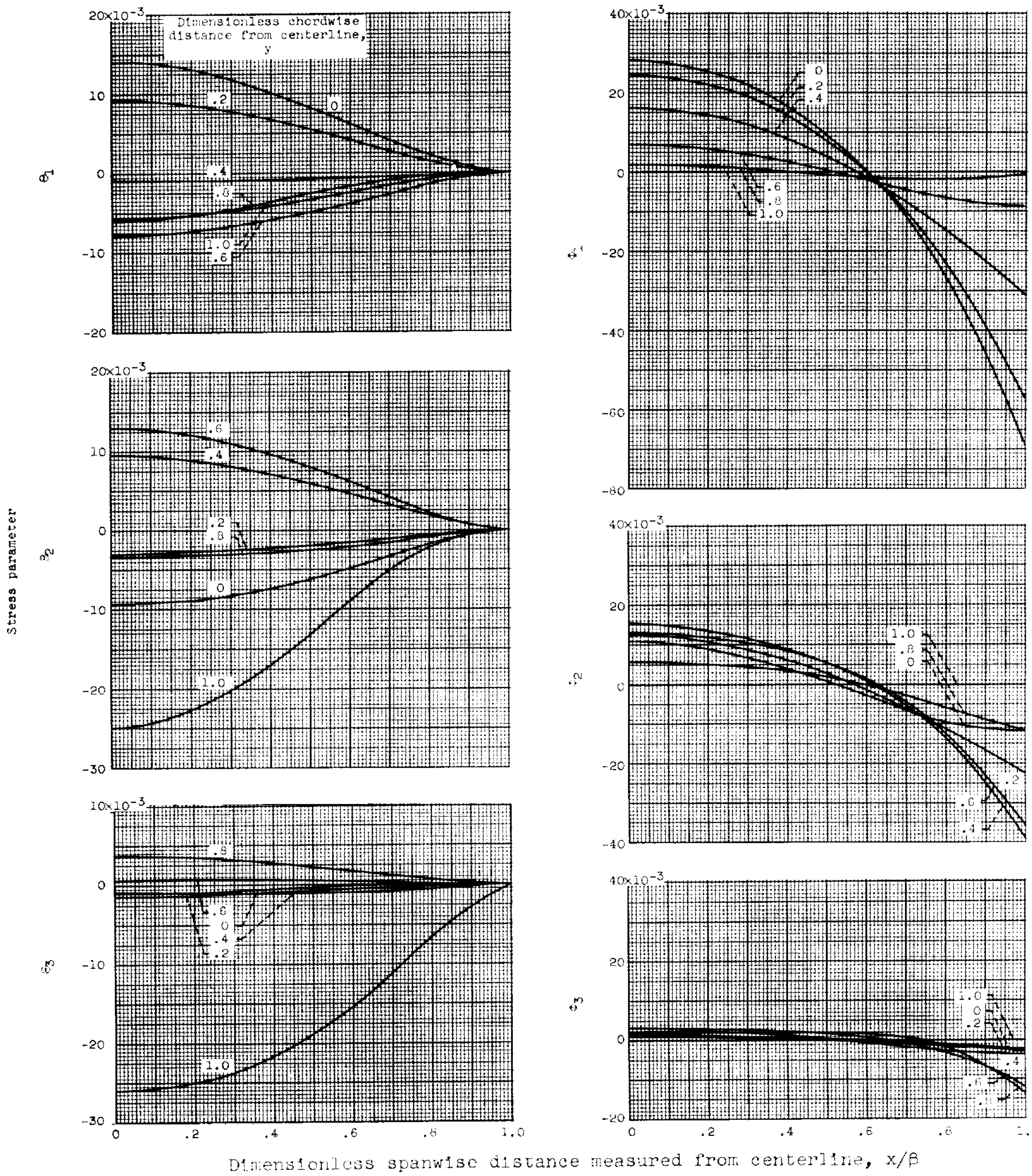


Figure 9. - Stress due to even thermal gradient for span-chord ratio $\beta = 0.50$.

E-1338

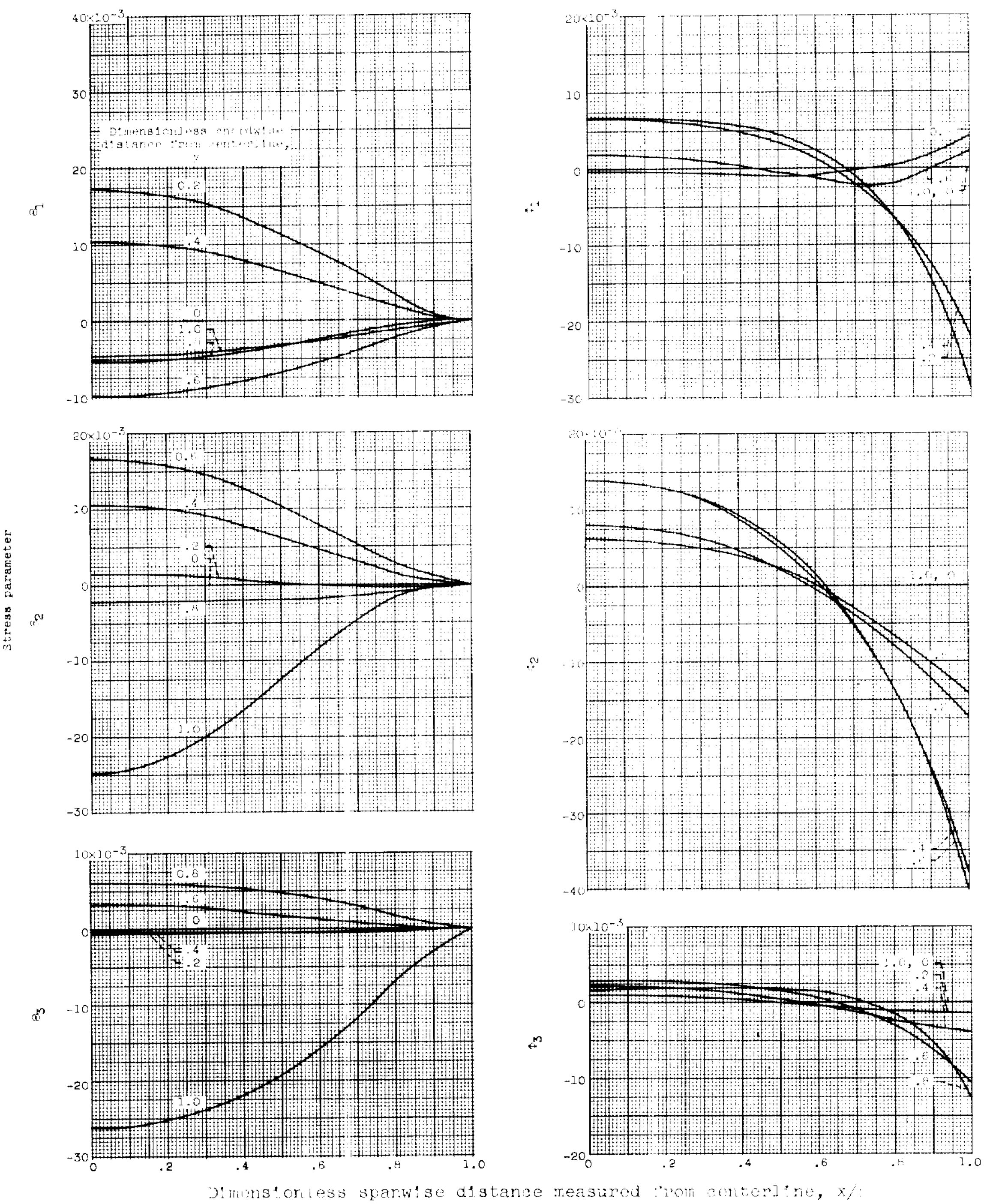


Figure 10. - Stress due to end thermal gradient for span-chord ratio, p = 0..0

E-1338

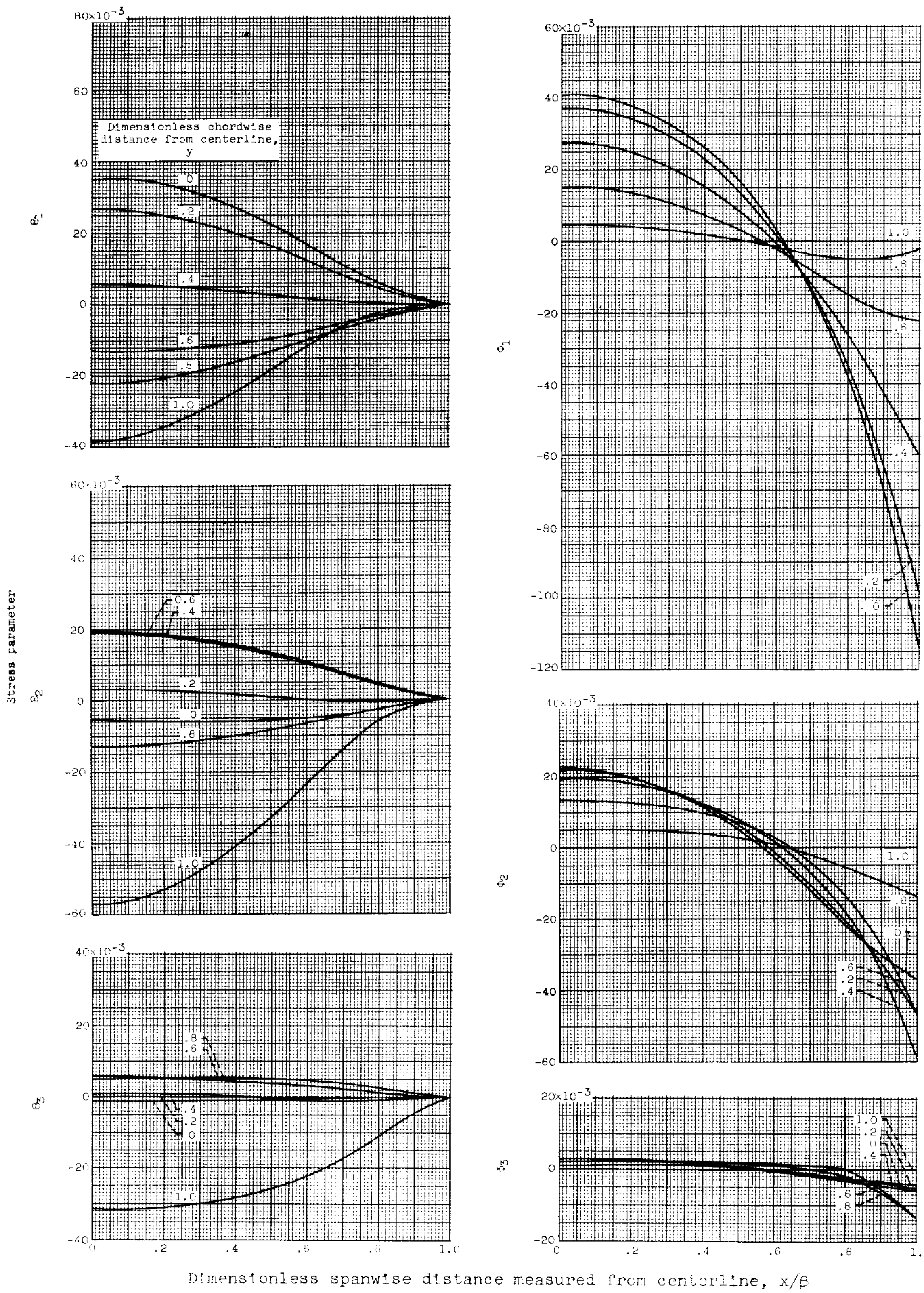


Figure 11. - Stress due to even thermal gradient for span-chord ratio $\beta = 0.75$.

T

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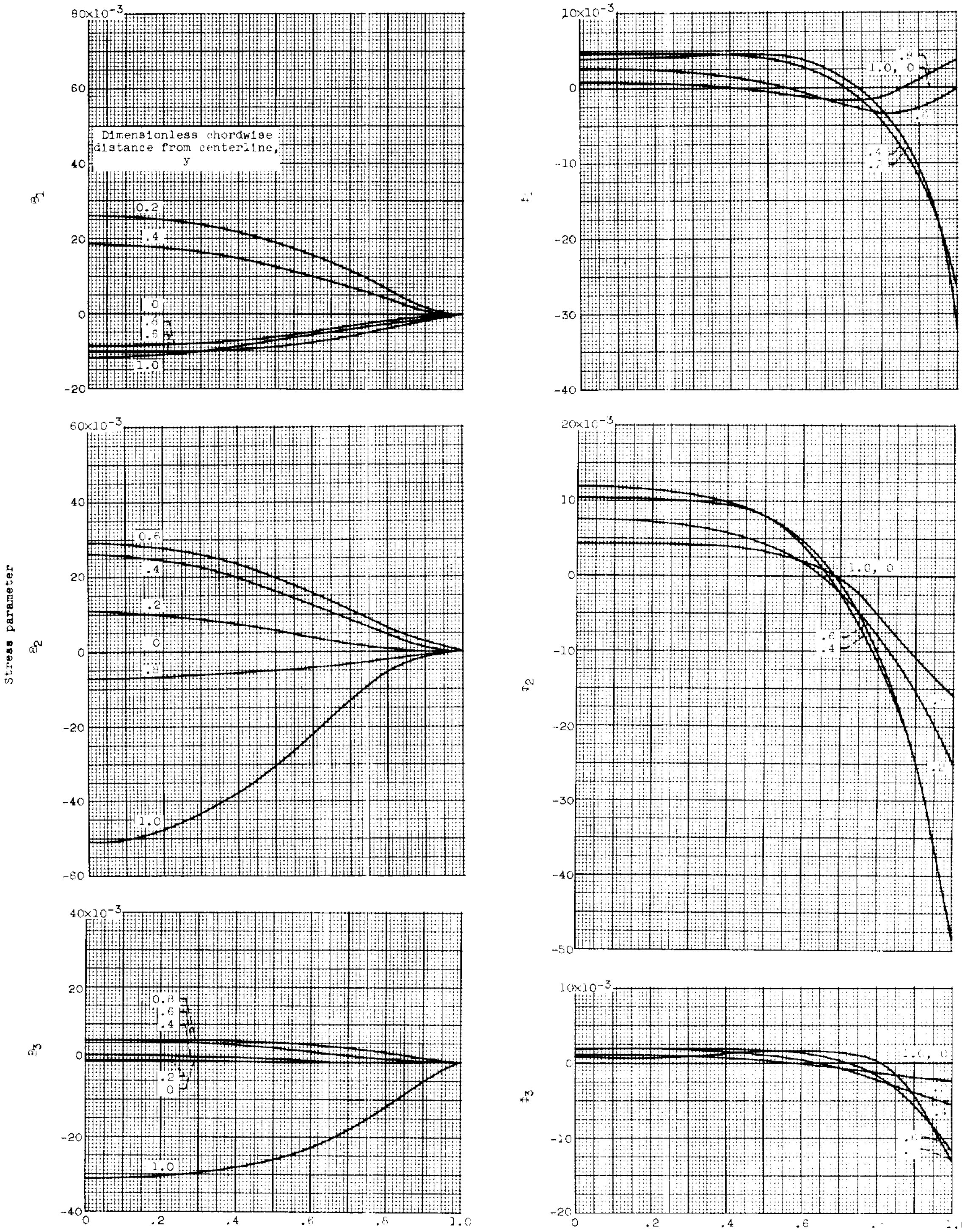


Figure 12. - Stress due to odd thermal gradient for span-chord ratio, p = 0.[illegible].

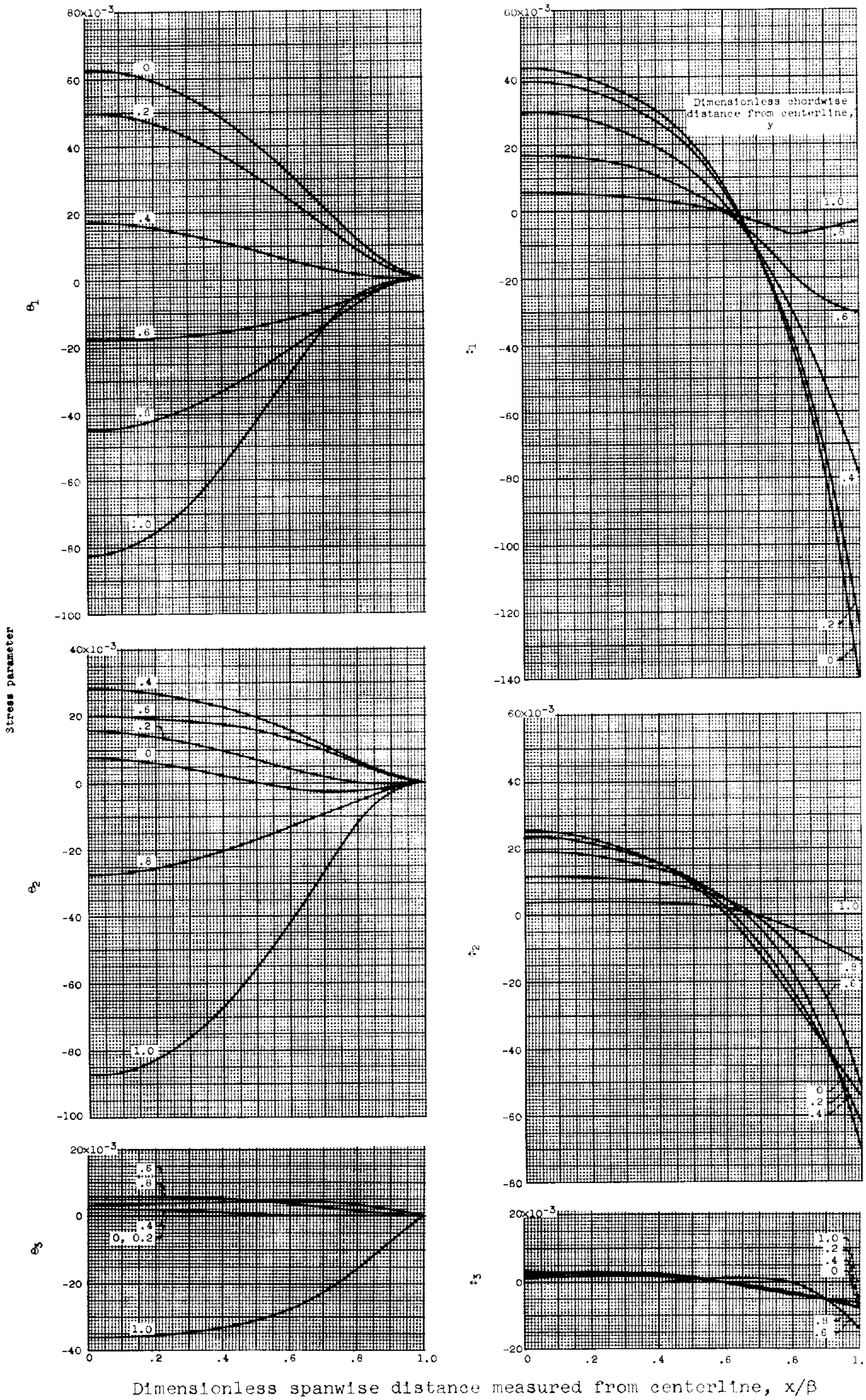


Figure 13. - Stress due to even thermal gradient for span-chord ratio β = 1.00.

E-1338

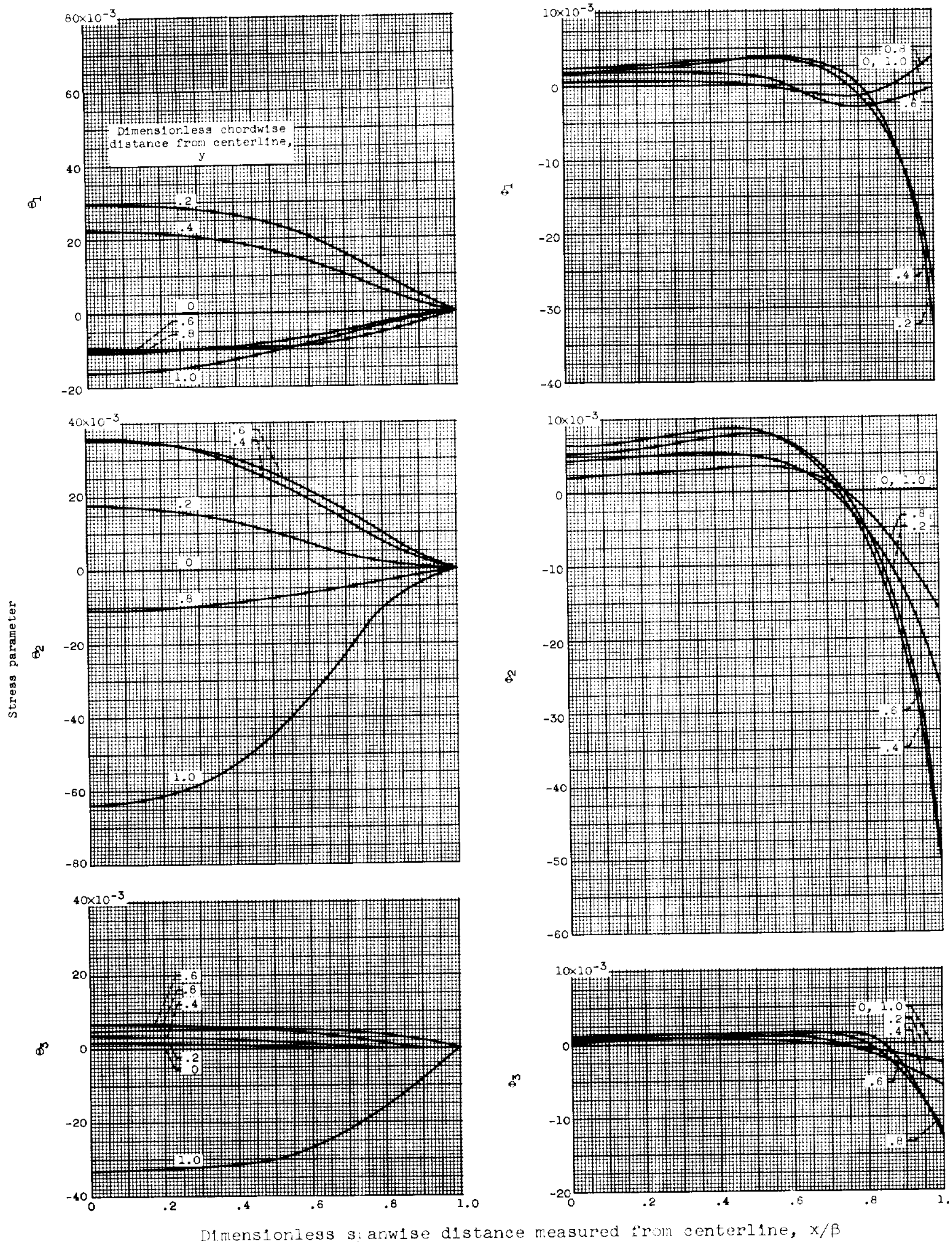


Figure 14. - Stress due to odd thermal gradient for span-chord ratio β = 1.00.

E-1338

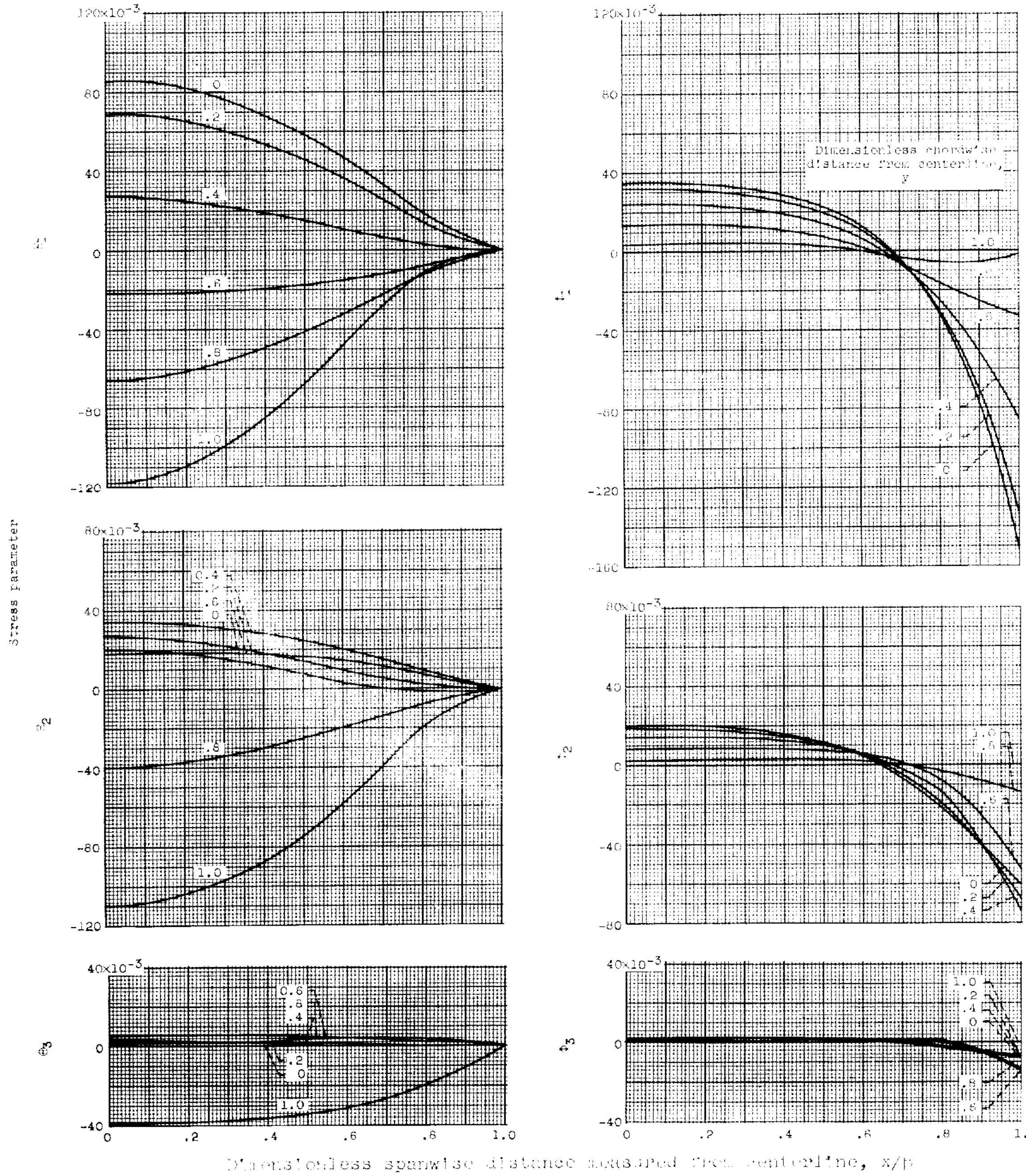
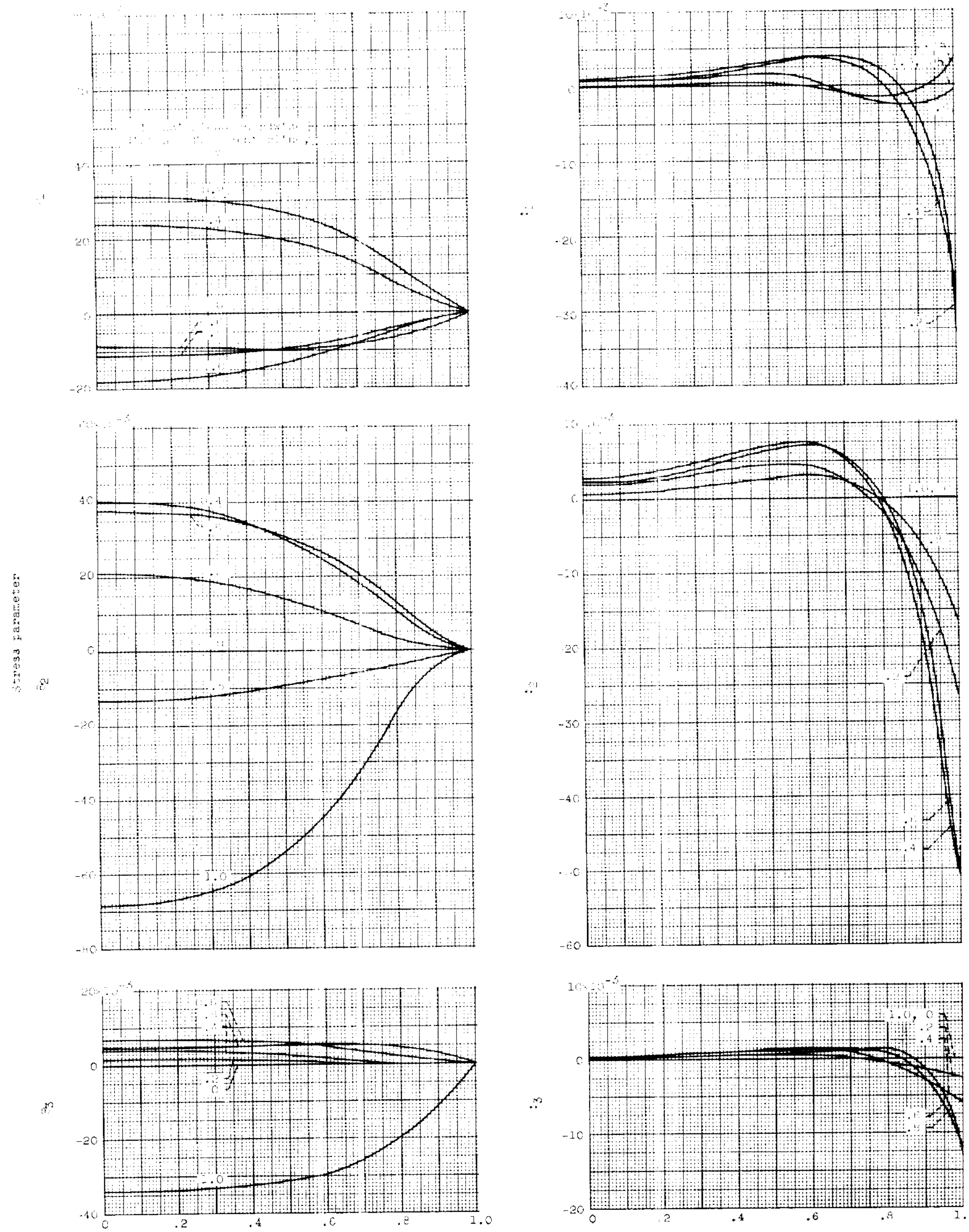


Figure 1[illegible]. - Stress due to even thermal gradient for span-chord ratio ρ = 1.2[illegible] .

E-1338



Dimensionless spanwise distance measured from centerline, x/P

Figure 16. - Stress due to edge thermal gradient for span-chord ratio ... = 1.2.

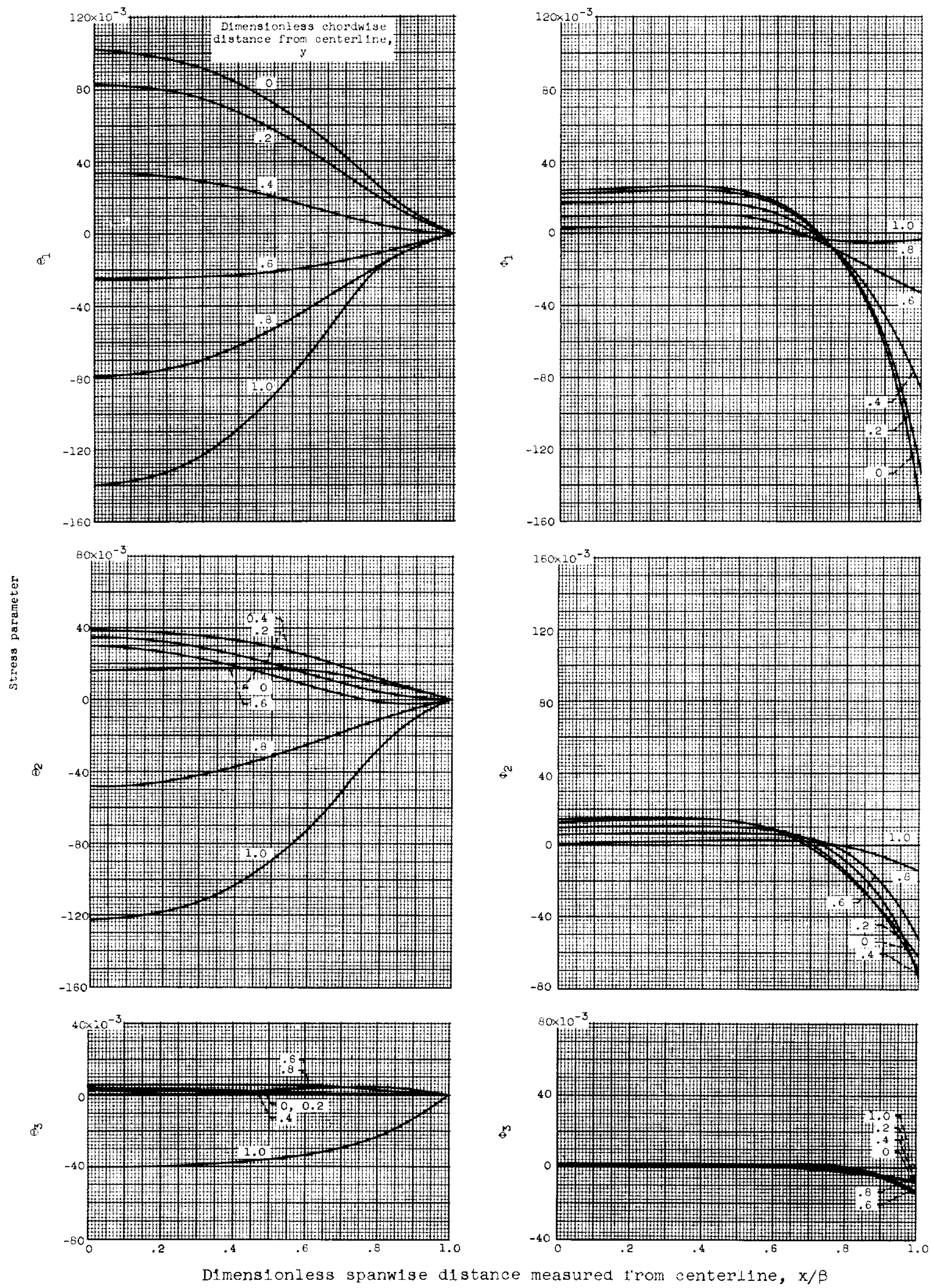


Figure 17. - Stress due to even thermal gradient for span-chord ratio β = 1.50.

E-1338

E-1338

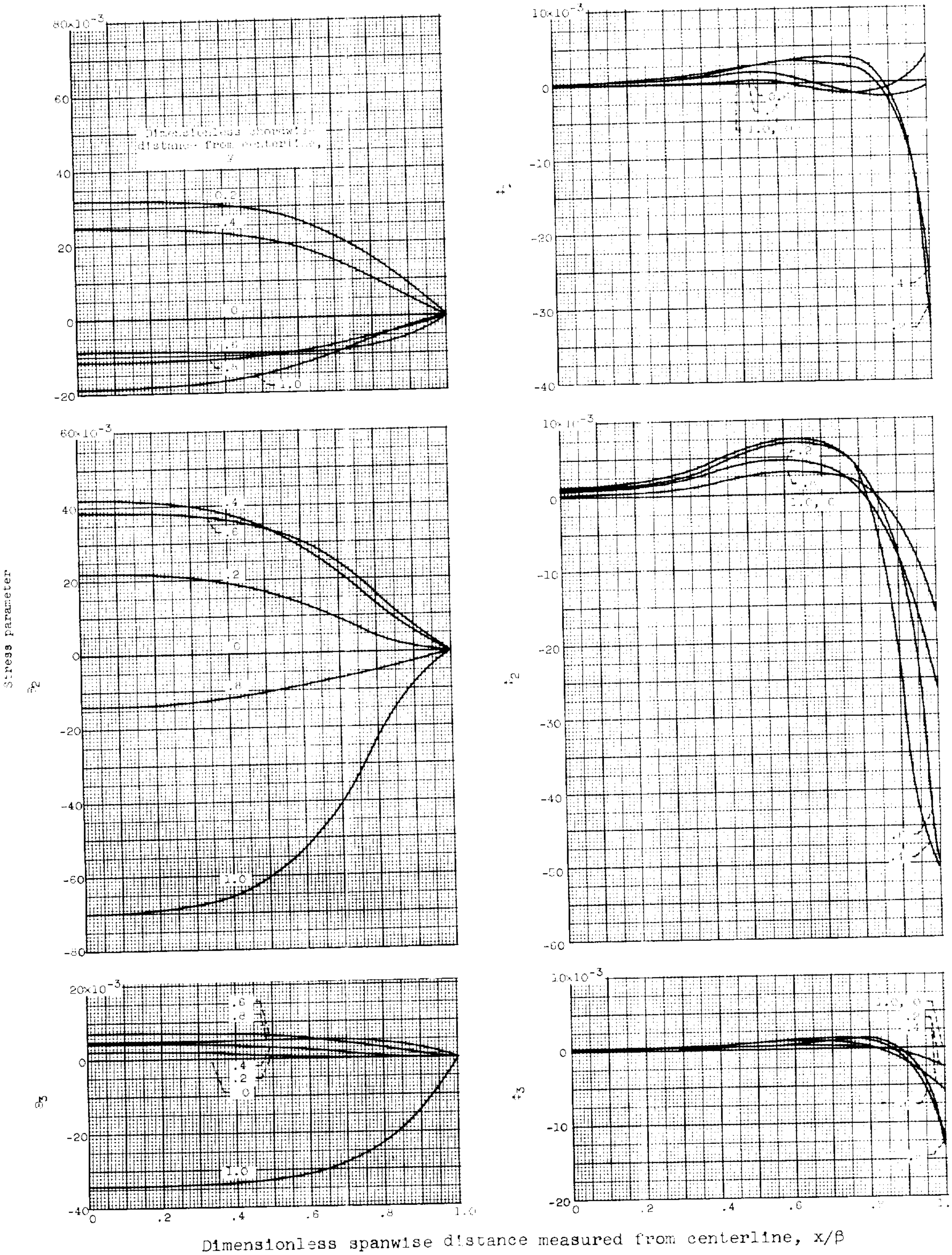


Figure 18. - Stress due to odd thermal gradient for span-chord ratio β = 1.50.

E-1338

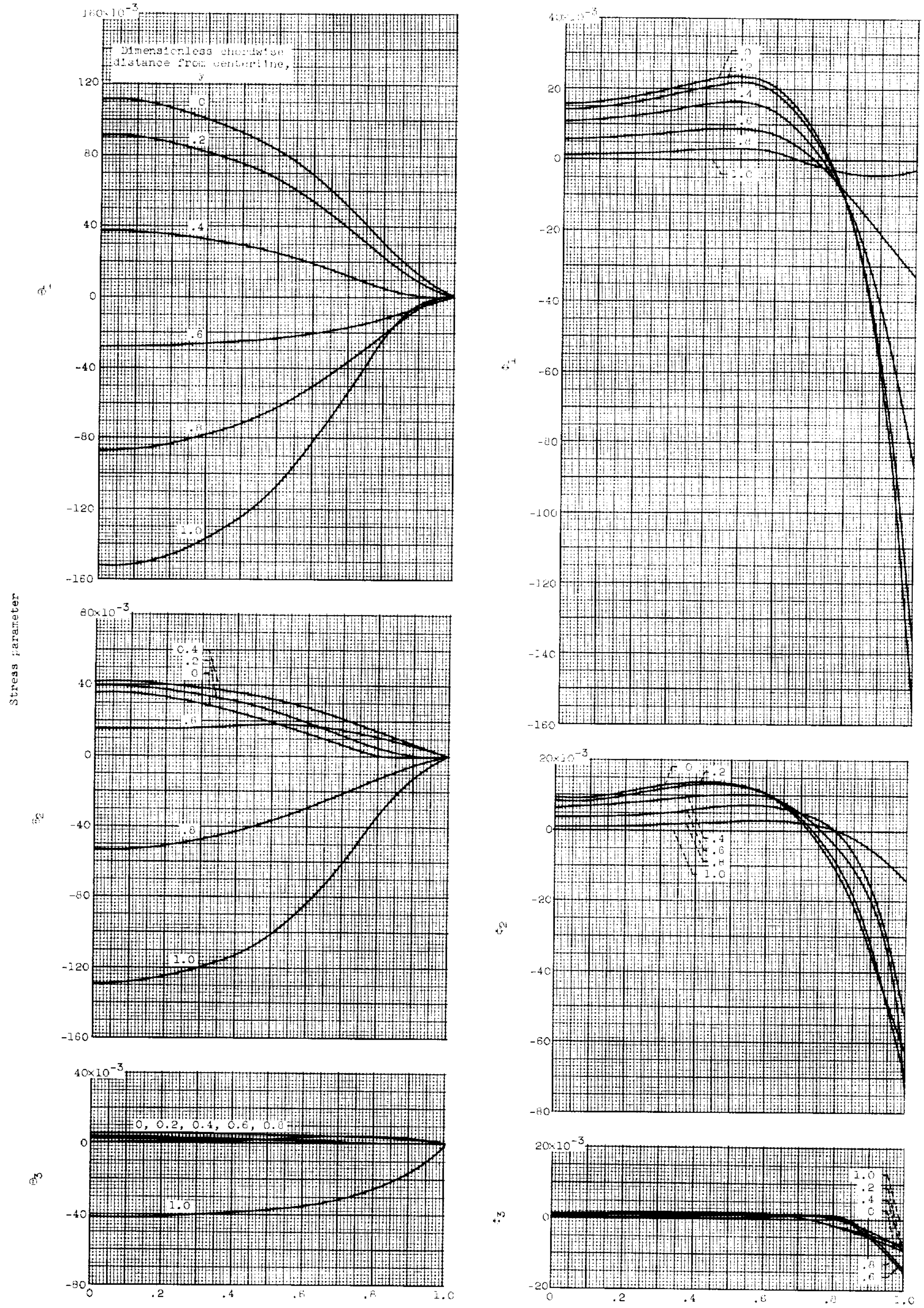


Figure 19. - Stress due to even thermal gradient for span-chord ratio $\beta = 1.75$.

E-1338

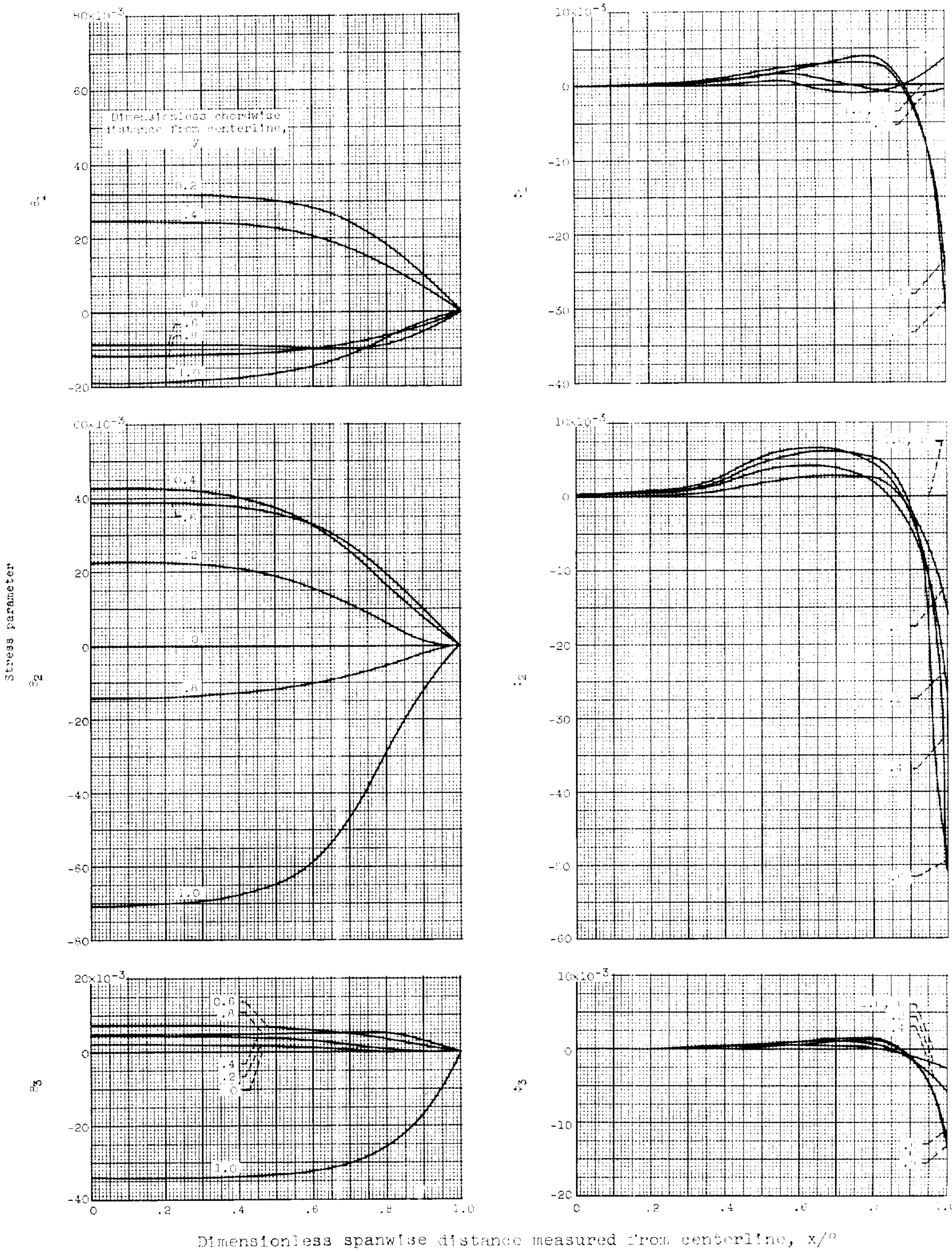


Figure 20. - Stress due to odd thermal gradient for span-chord ratio ρ = 1.7[illegible].

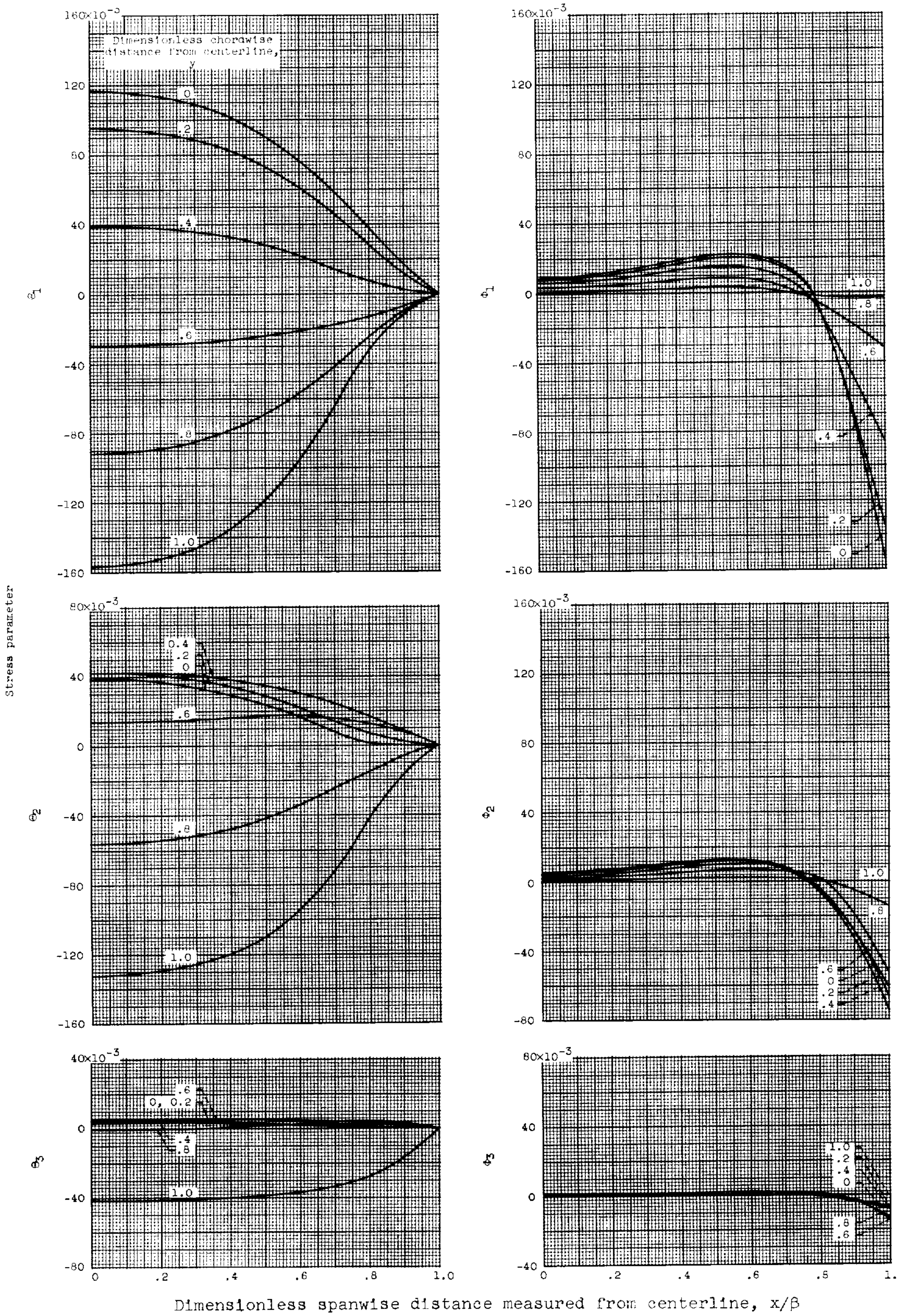


Figure 21. - Stress due to even thermal gradient for span-chord ratio β = 2.00.

E-1338

E-1338

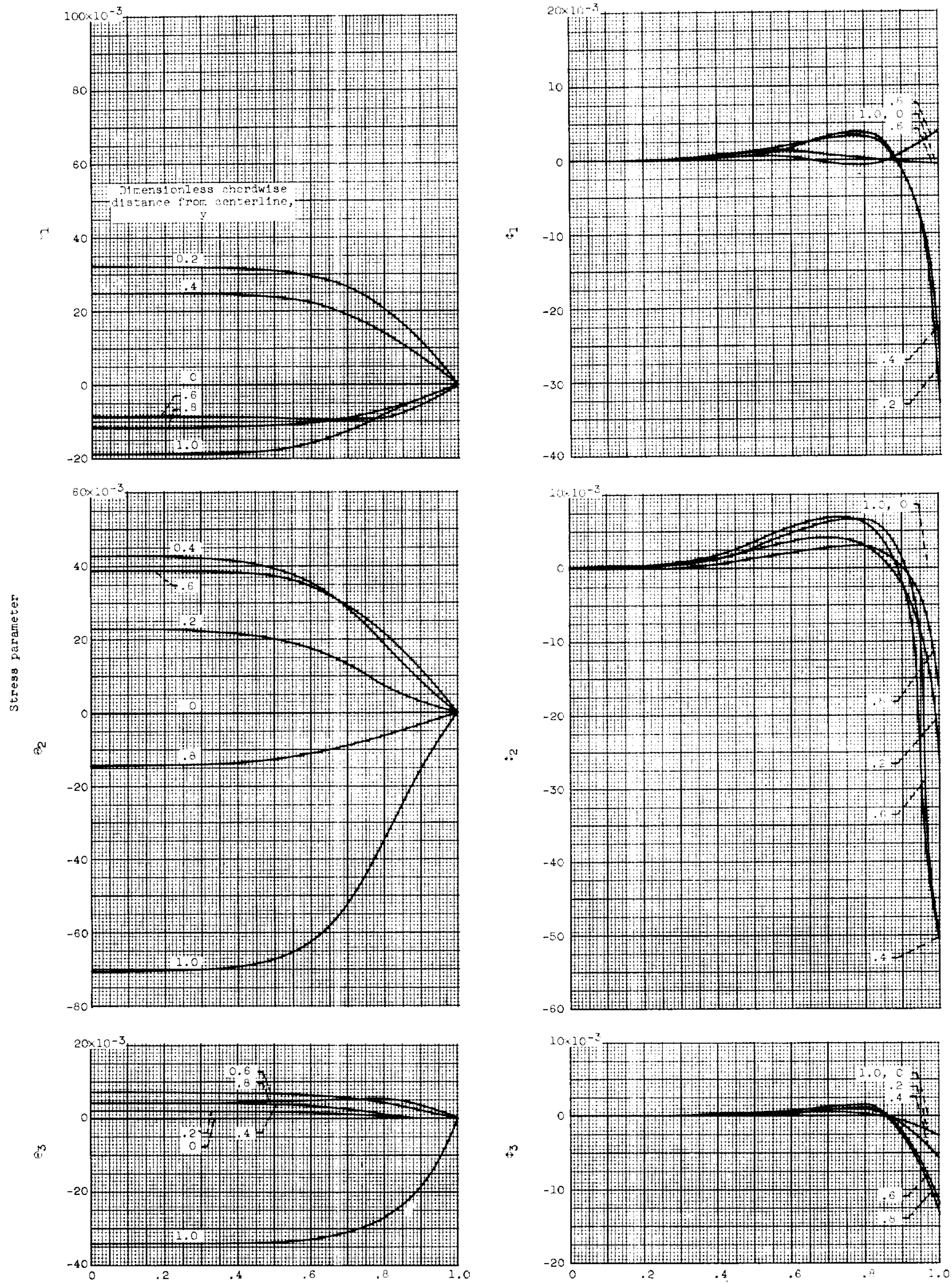


Figure 22. - Stress due to odd thermal gradient for span-chord ratio β = 2.00.

E-1338

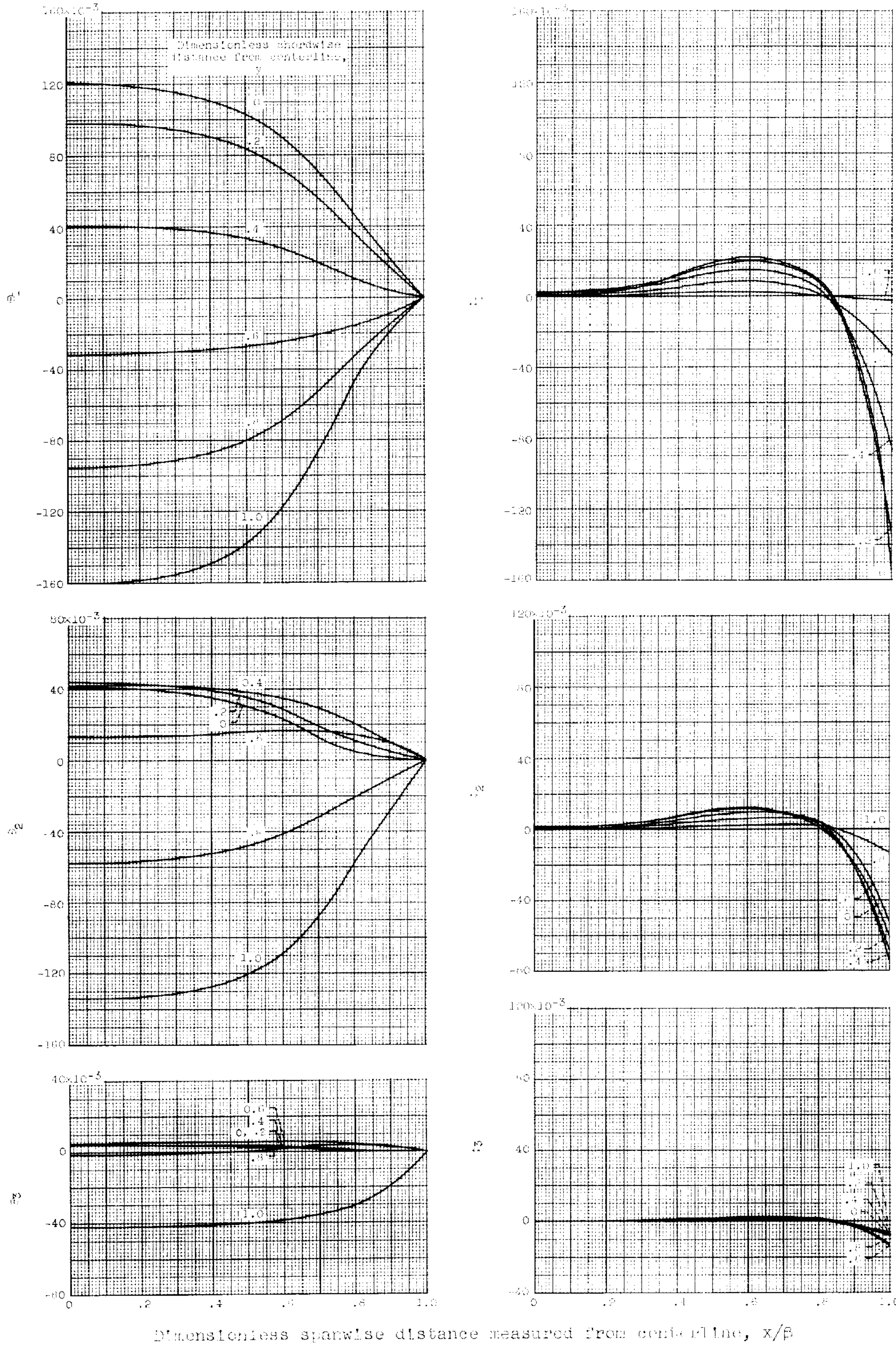
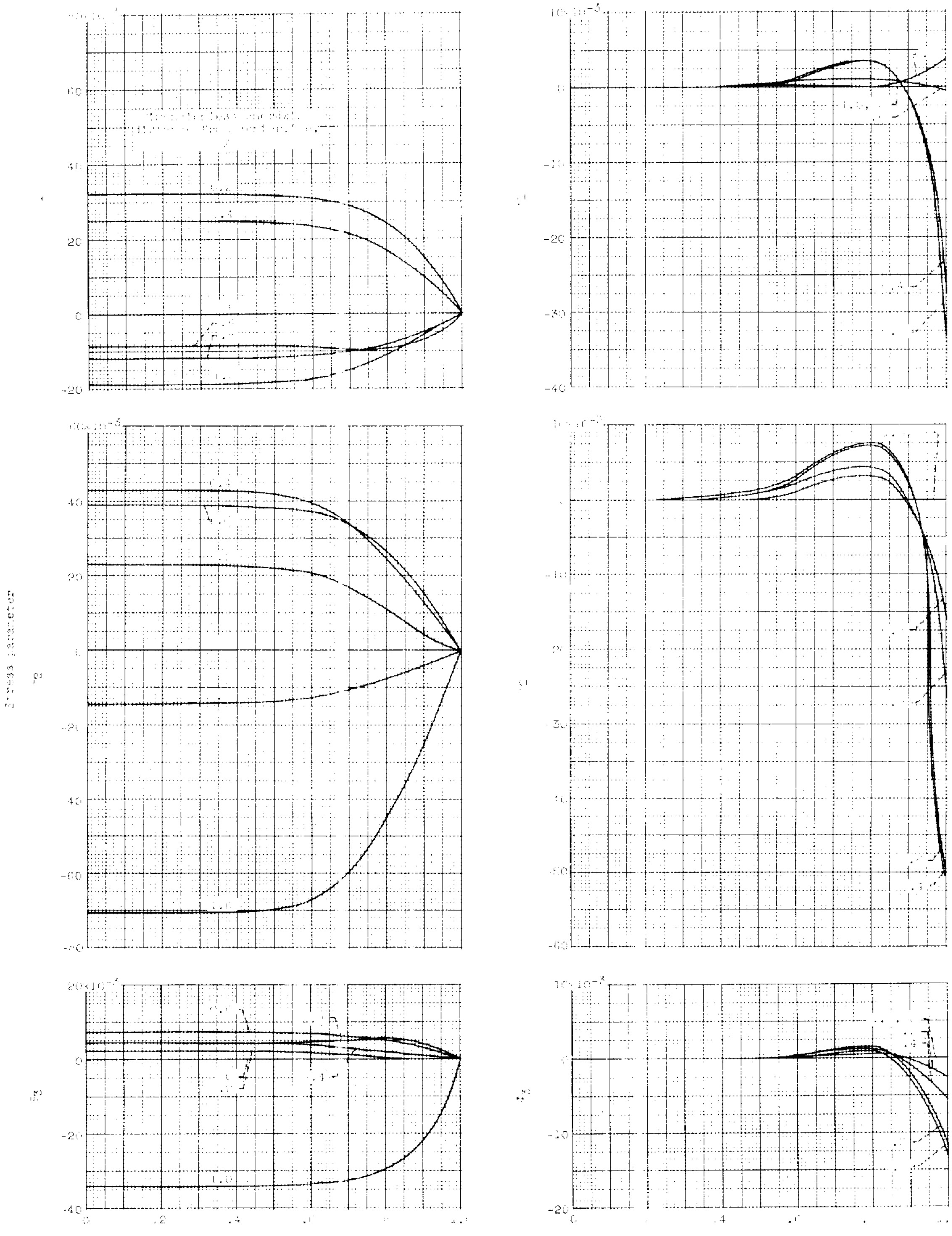


Figure 23. - Stress due to even thermal gradient for span-chord ratio β = 2.50.

E-1338



Dimensionless spanwise distance measured from centerline, x/β

Figure 21. - Stress due to odd thermal gradient for span-chord ratio β = 2.50.

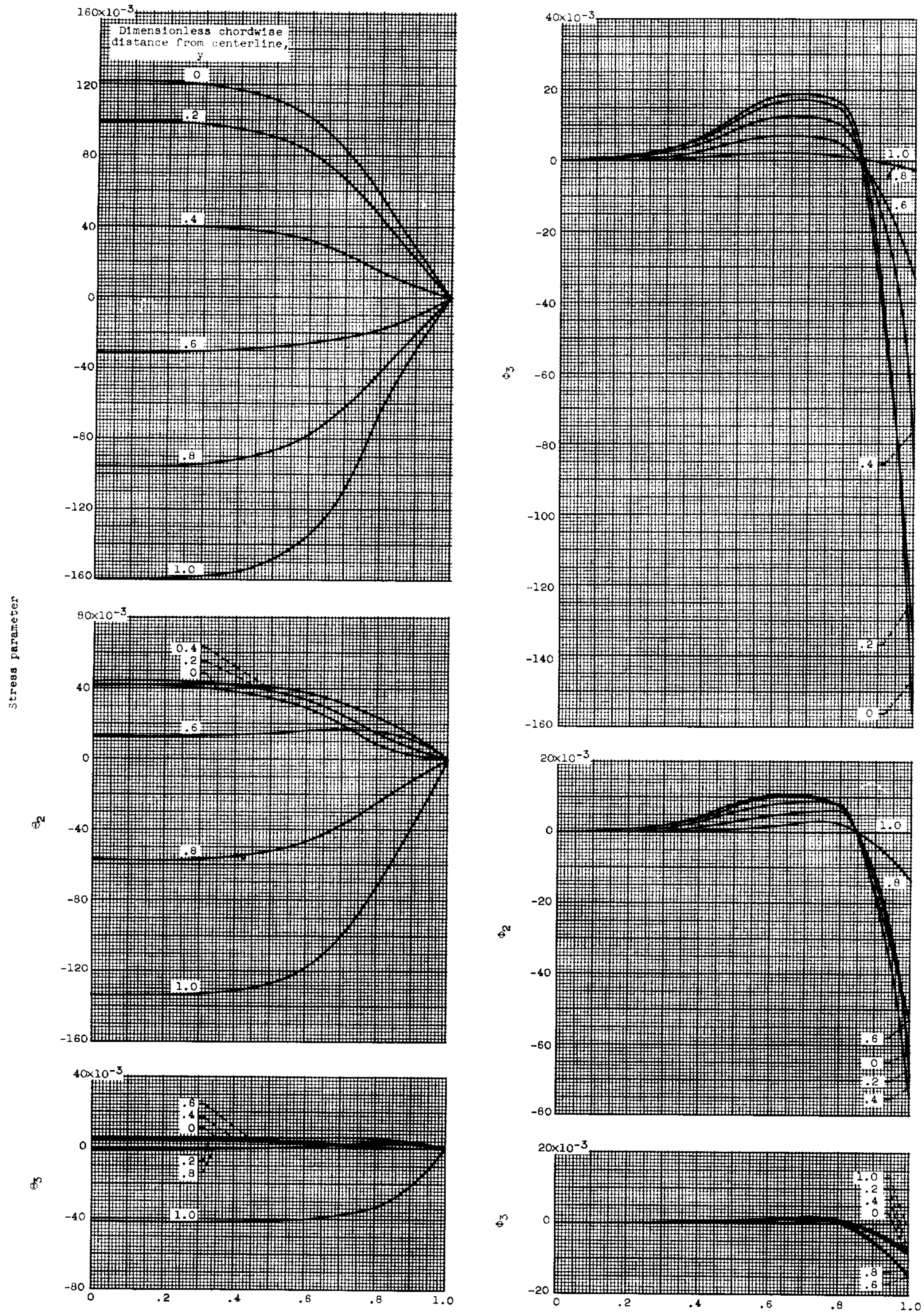


Figure 25. - Stress due to even thermal gradient for span-chord ratio $\beta = 3.00$.

E-1338

E-1338

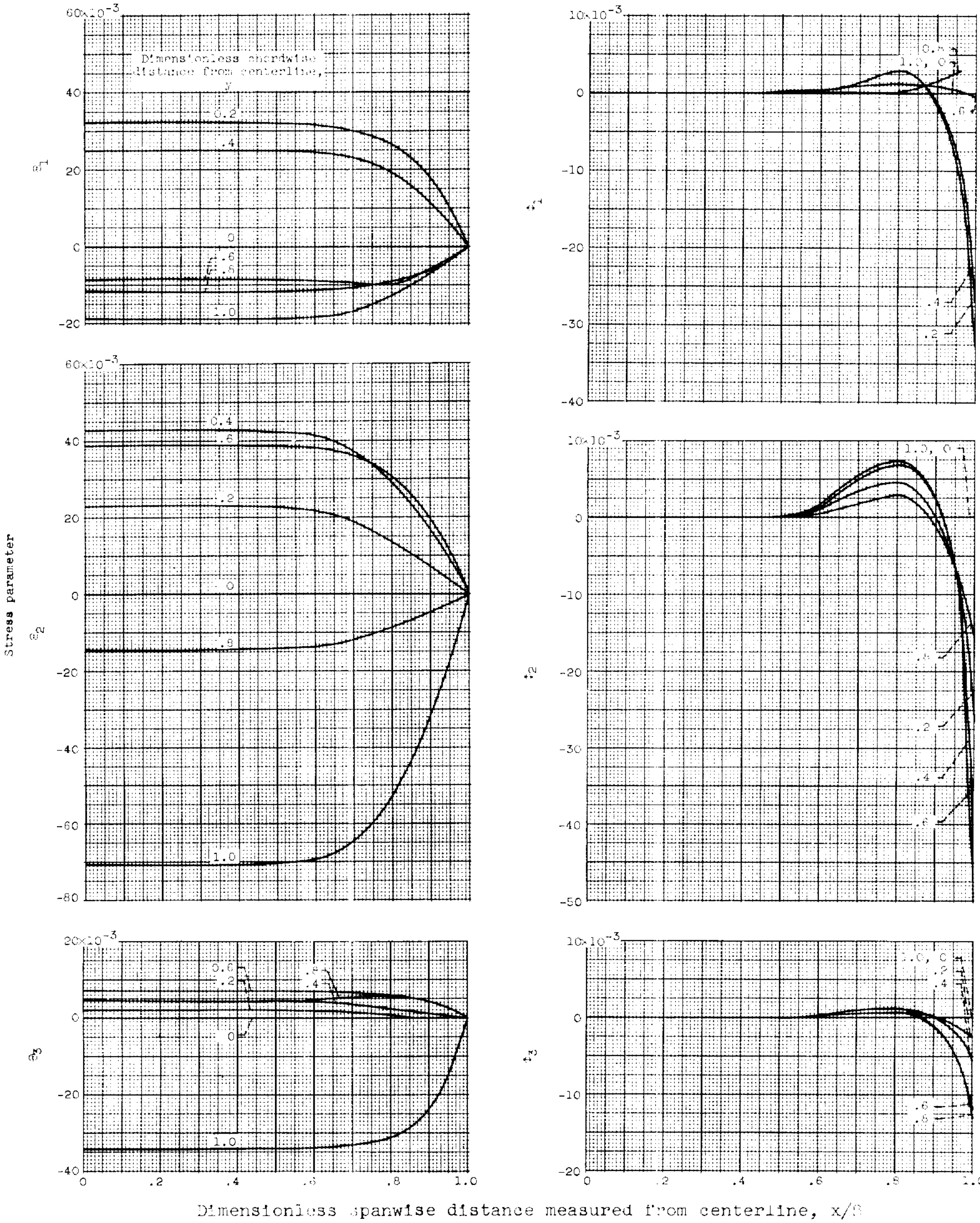
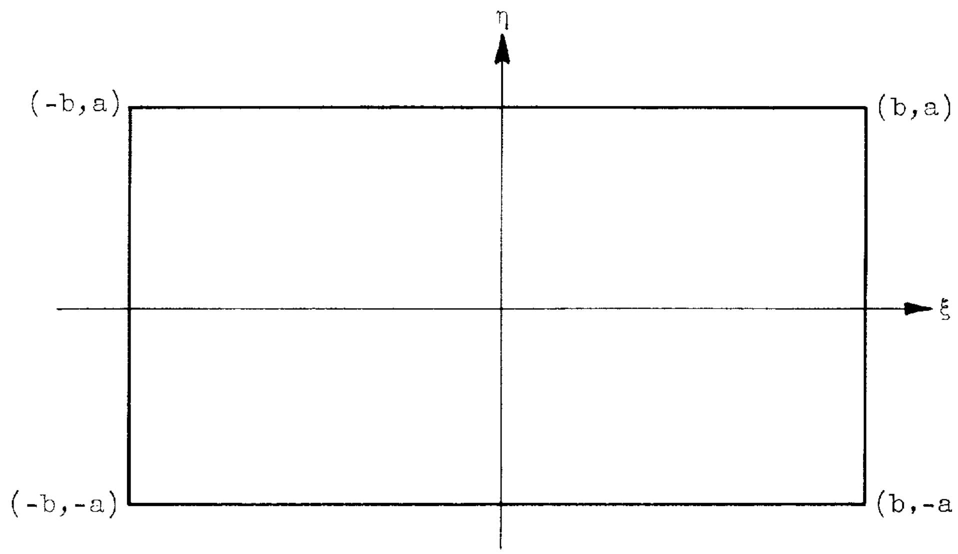
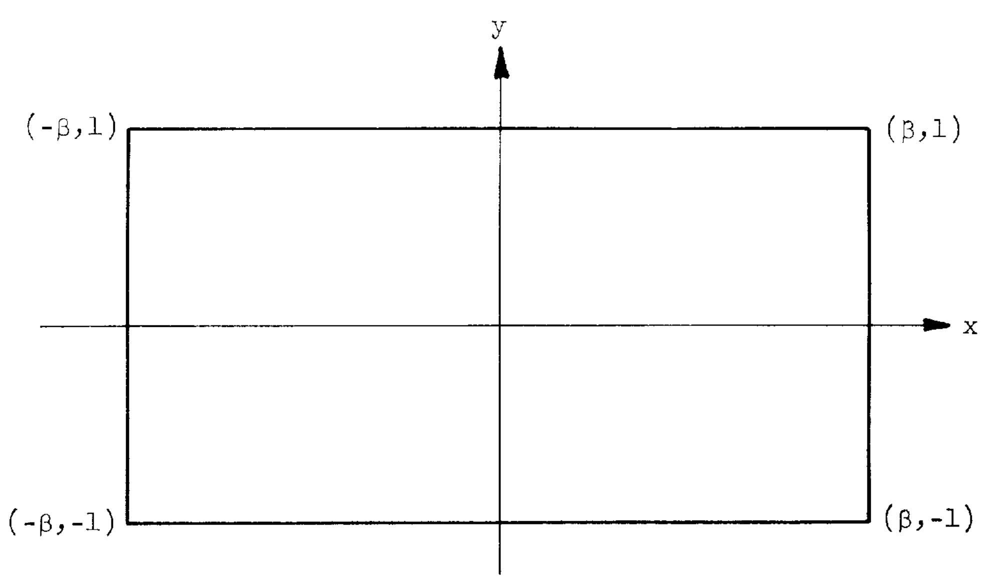


Figure 26. - Stress due to odd thermal gradient for span-chord ratio $\beta = 3.00$.



(a) ξ,η coordinate system.



(b) x,y coordinate system.

Figure 27. - Coordinate systems used to calculate stresses developed in thermally loaded, thin flat plate.

E-1338

E-1338

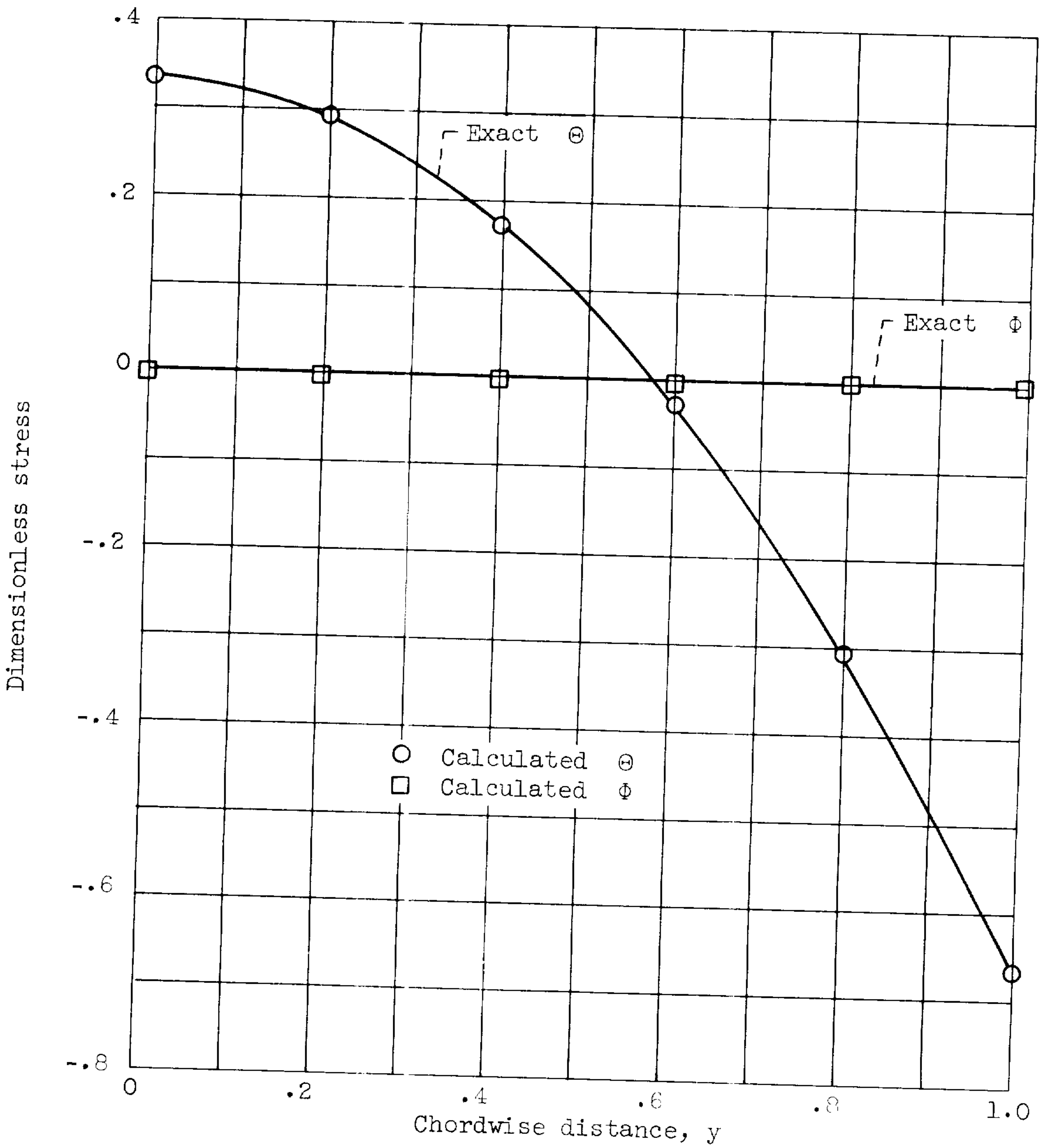


Figure 28. - Comparison of calculated and exact dimensionless stresses along chord far from end of semi-infinite plate for parabolic chordwise temperature distribution; $T = \left(y^2 - \frac{1}{3}\right)T_o$.

E-1338

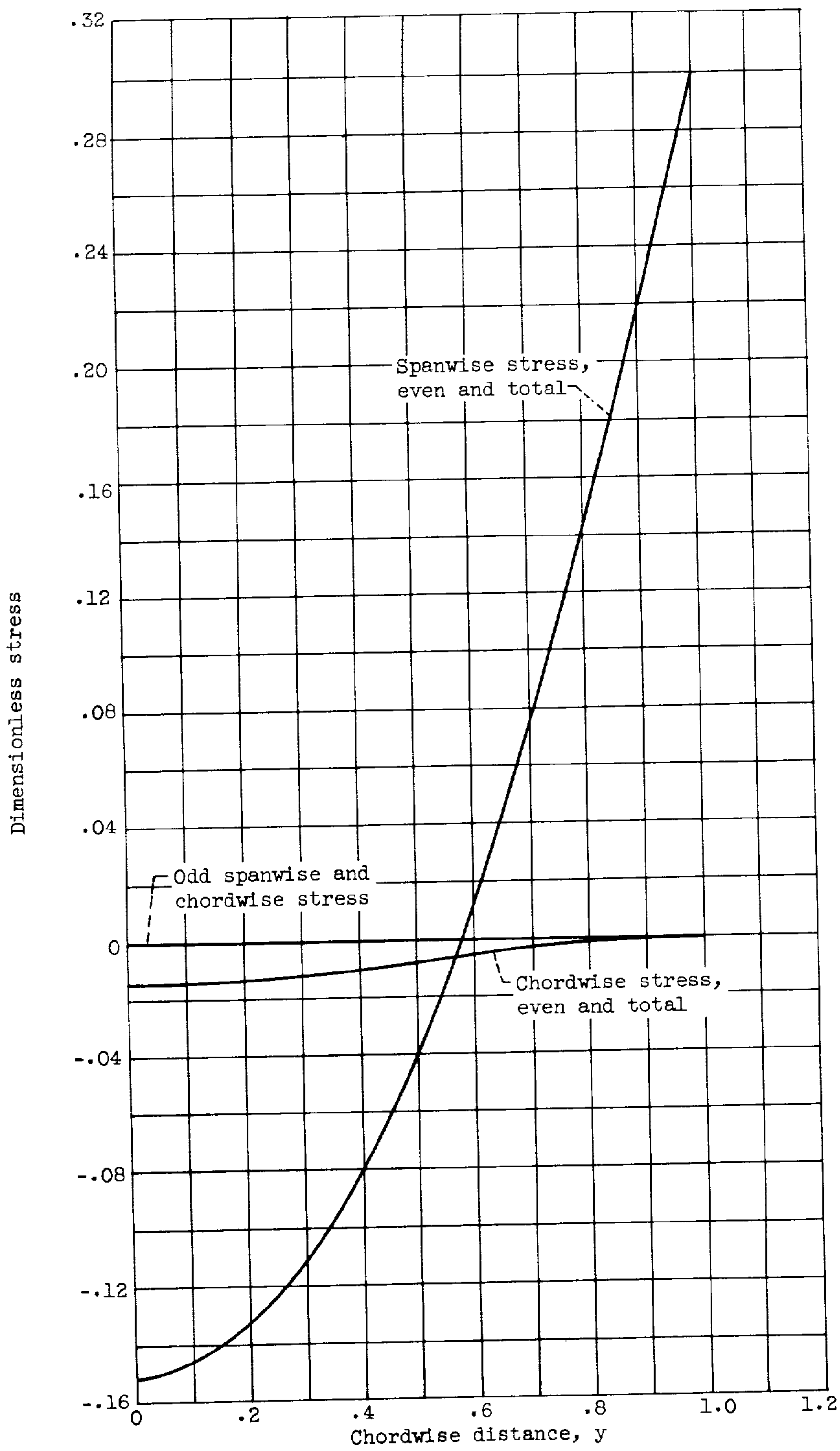


Figure 29. - Stresses along semichord at center of 2 by 1 plate; $T/T_o = \sin(5x) + \cos(y)$.

E-1338

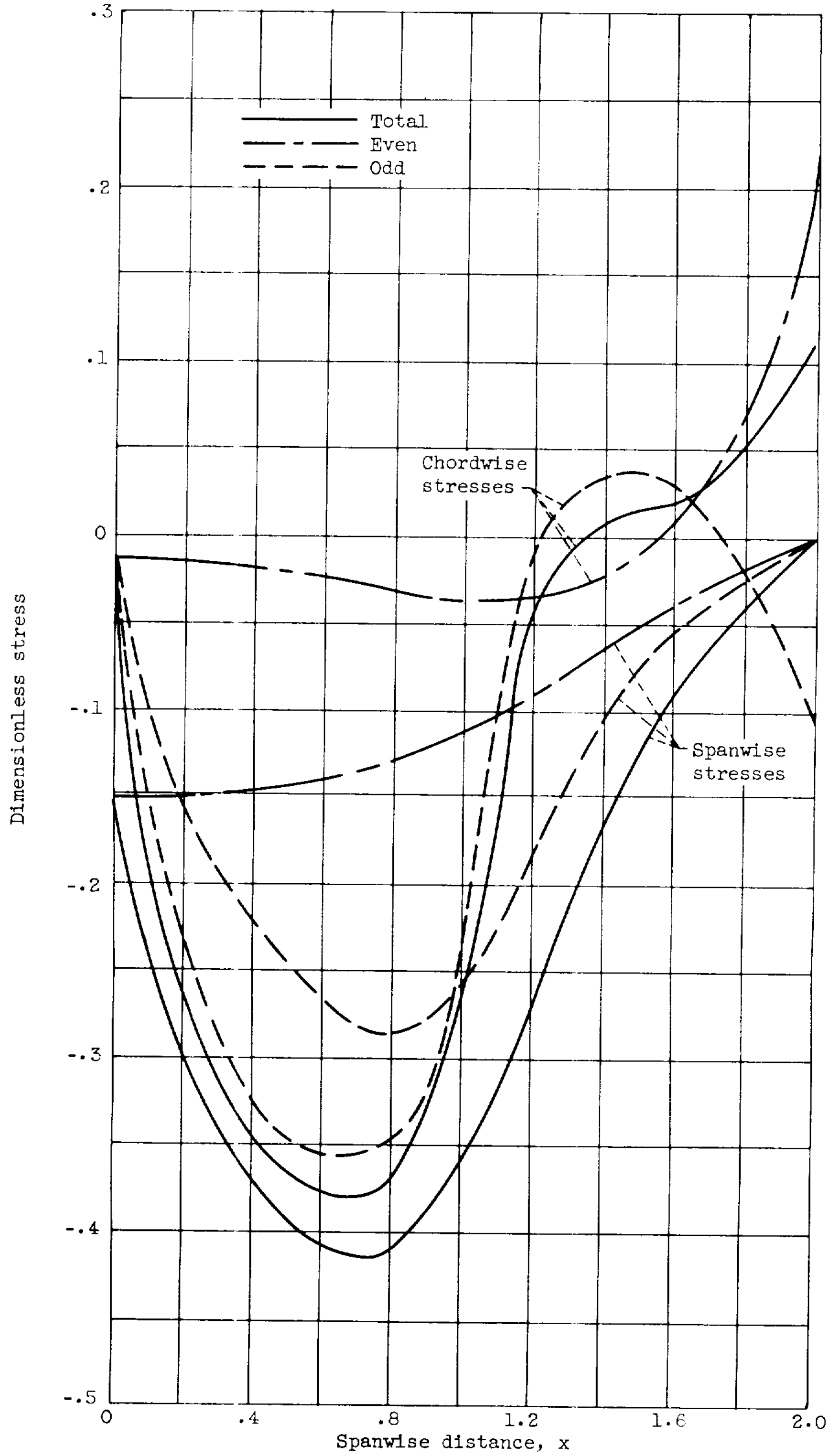


Figure 30. - Stresses along semispan at center of 2 by 1 plate; $T/T_o = \sin(5x) + \cos(y)$.

E-1338

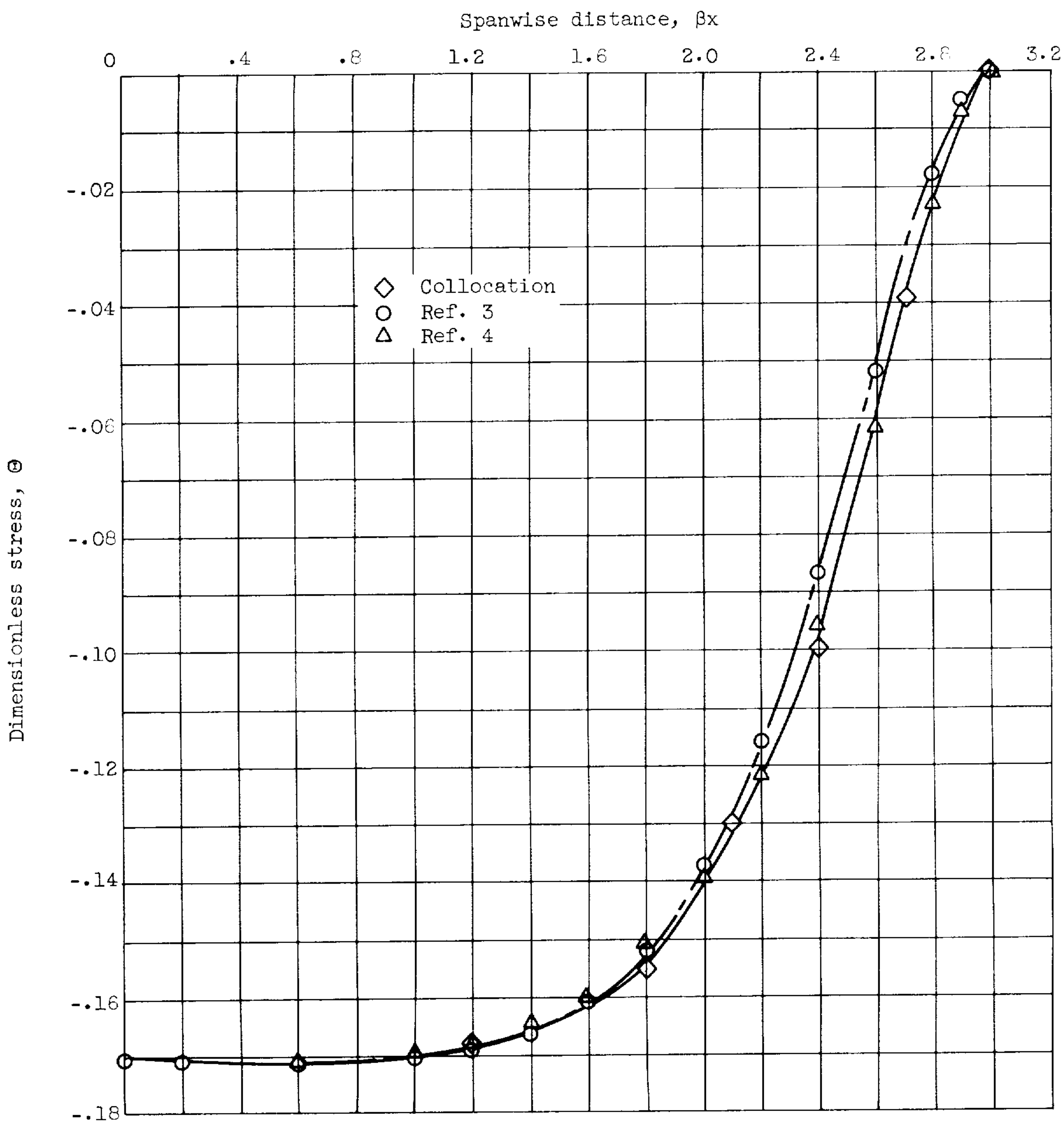


Figure 31. - Plot of Θ against x at $y = -0.9$ for a 3 by 1 plate; $T/T_o = 1.014 - 0.08940\ y + 0.001074\ (1 - y)^9$.

E-1338

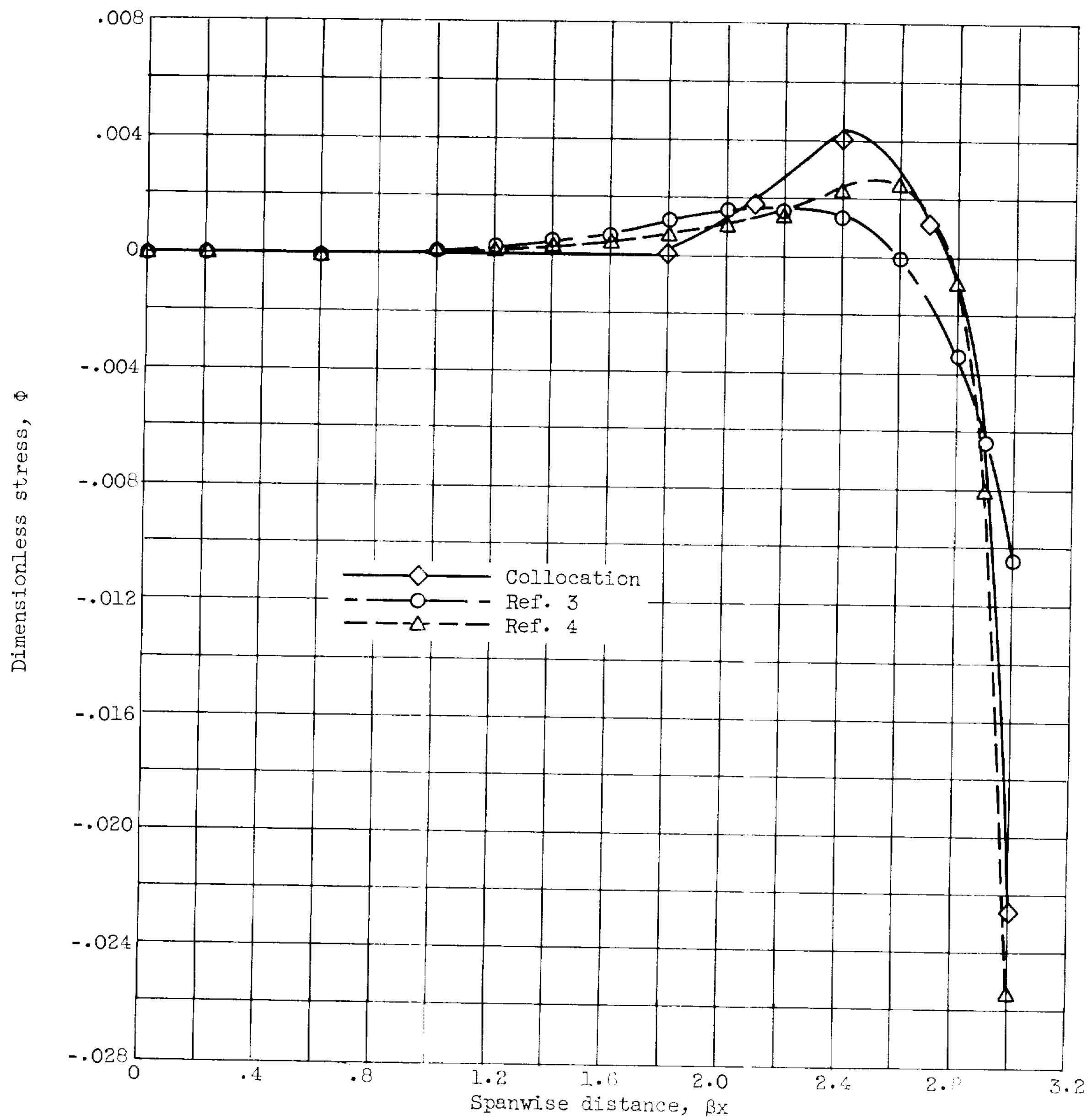


Figure 32. - Plot of Φ against x at $y = -0.9$ for a 3 by 1 plate; $T/T_o = 1.014 - 0.08340\ y + 0.001074\ (1 - y)^9$.

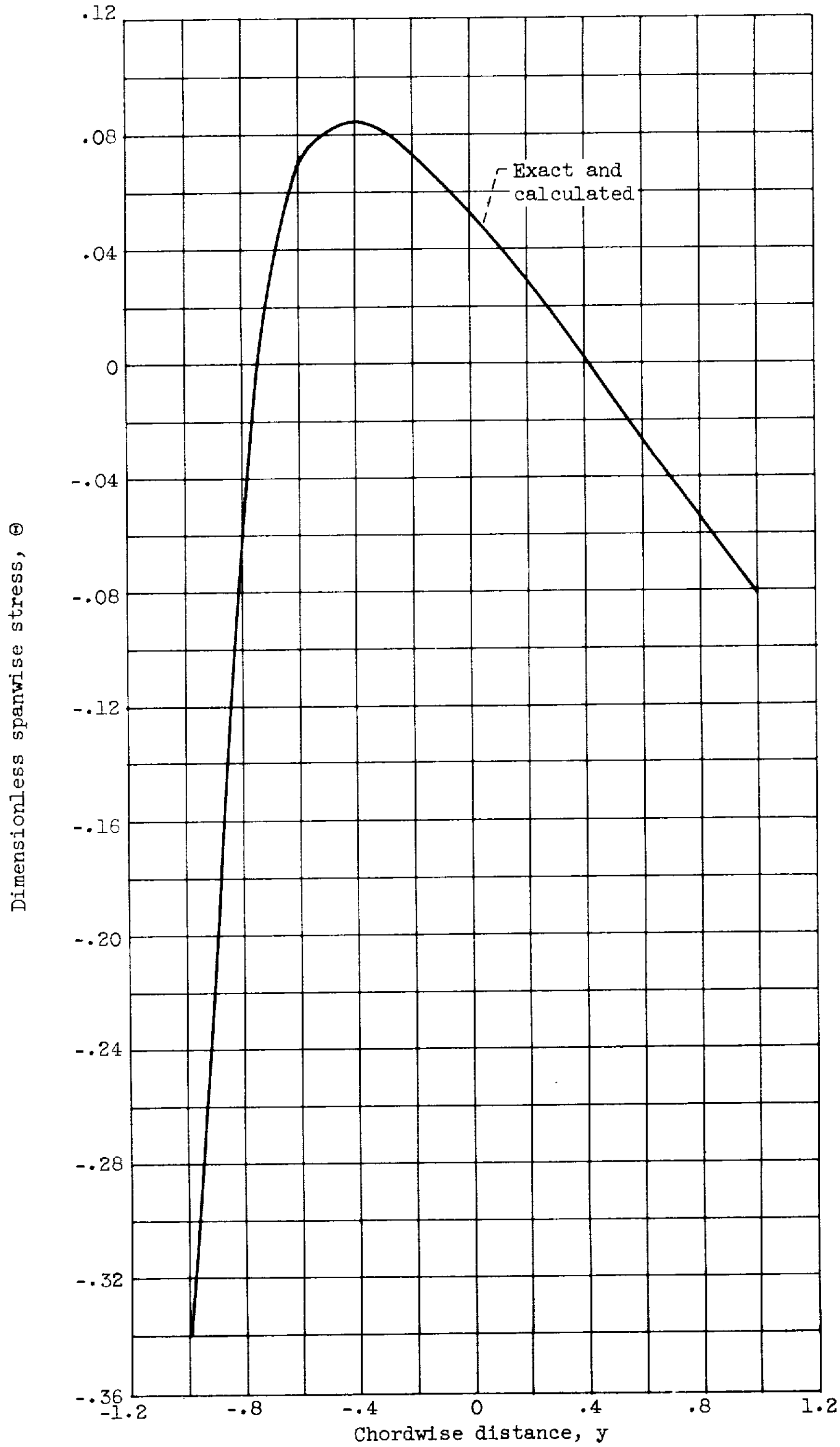


Figure 33. - Plot of Θ against y at $x = 0$ for a 3 by 1 plate, exact and collocation methods; $T/T_o = 1.014 - 0.08940\ y + 0.001074\ (1 - y)^9$.